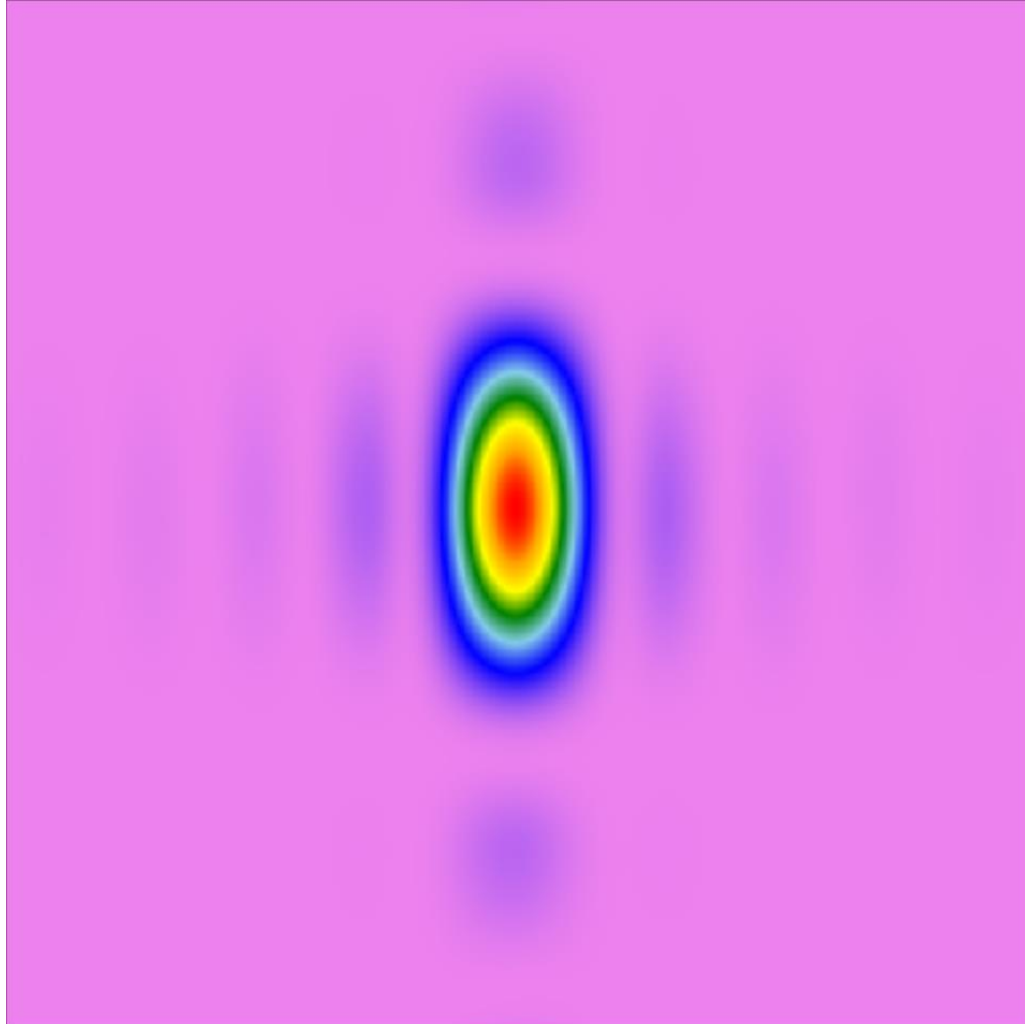


# Grazing-Incidence Focusing Mirrors for X-Ray Beams

# Abstract

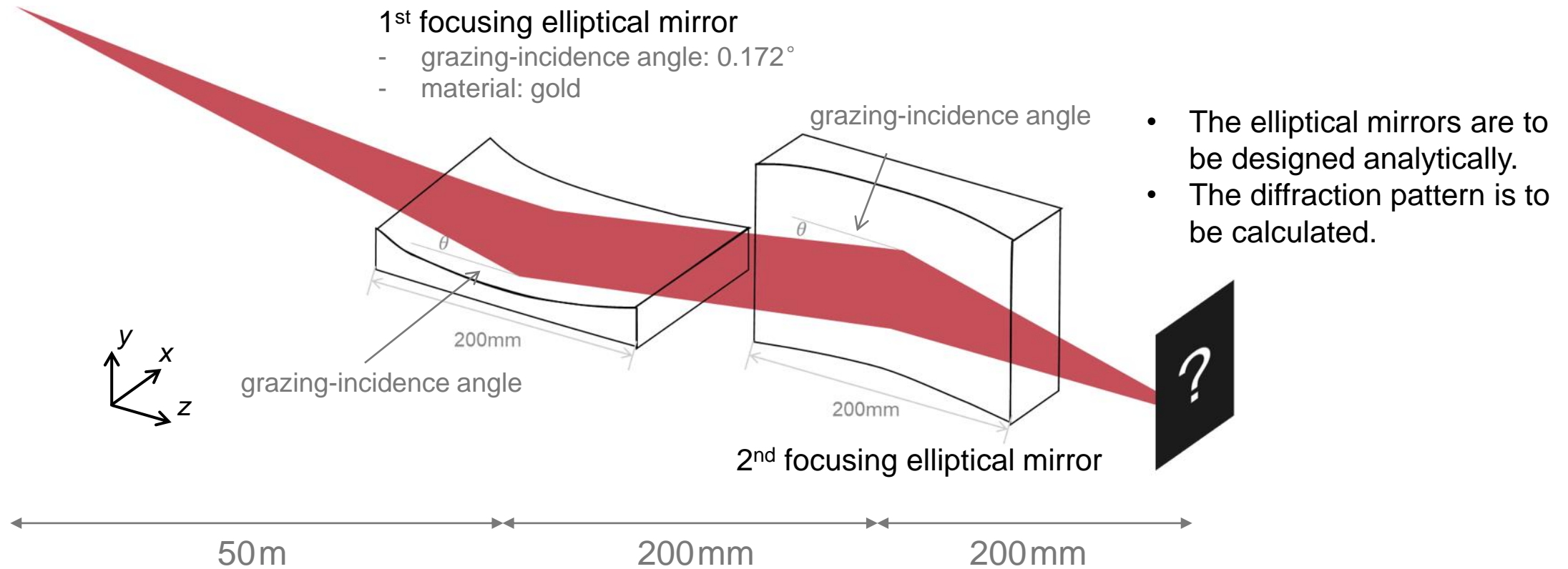


Grazing-incidence reflective optics are widely used in X-ray beamlines, in particular Kirkpatrick-Baez (KB) elliptical mirror systems [A. Verhoeven, *et al.*, *Journal of Synchrotron Radiation* 27.5 (2020): 1307-1319]. Focusing is accomplished by using two physically separated elliptical mirrors to focus the beam in two dimensions. The incoming X-rays can be focused by the system down to nanometer-scale spot size. Such system is modeled and simulated in VirtualLab Fusion and the focal field is calculated.

# Modeling Task

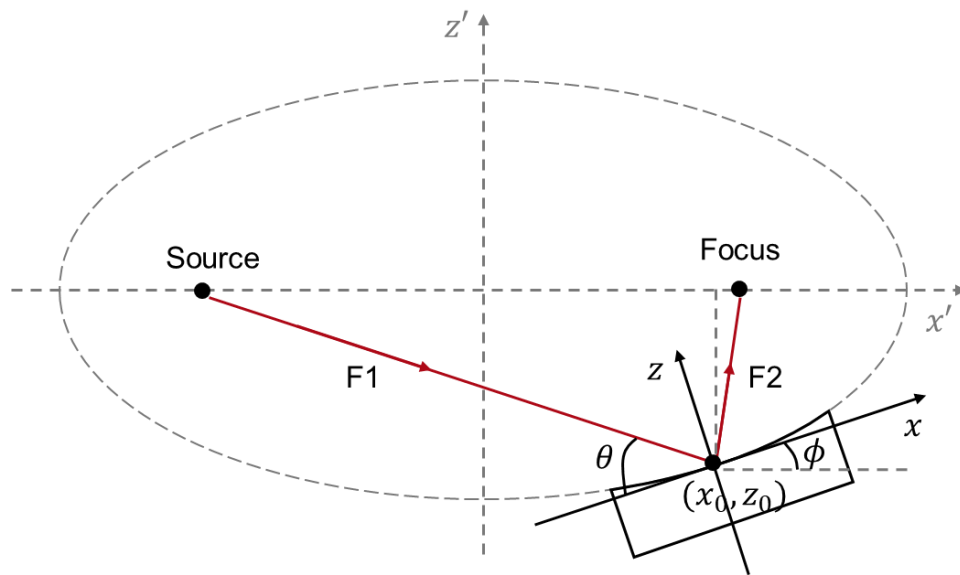
## fundamental Gaussian

- wavelength: 173pm
- waist radius:  $4.44\mu\text{m} \times 4.44\mu\text{m}$



system parameters from A. Verhoeven, et al., *Journal of Synchrotron Radiation* 27.5 (2020): 1307-1319.

# Analytical Design of the Elliptical Mirror (1)



For the calculation of the elliptical surface height profile  $z(x)$  the following parameters are required:

- distance between source and the mirror center  $F_1$
- distance between image/focus and the mirror center  $F_2$
- grazing-incidence angle  $\theta$

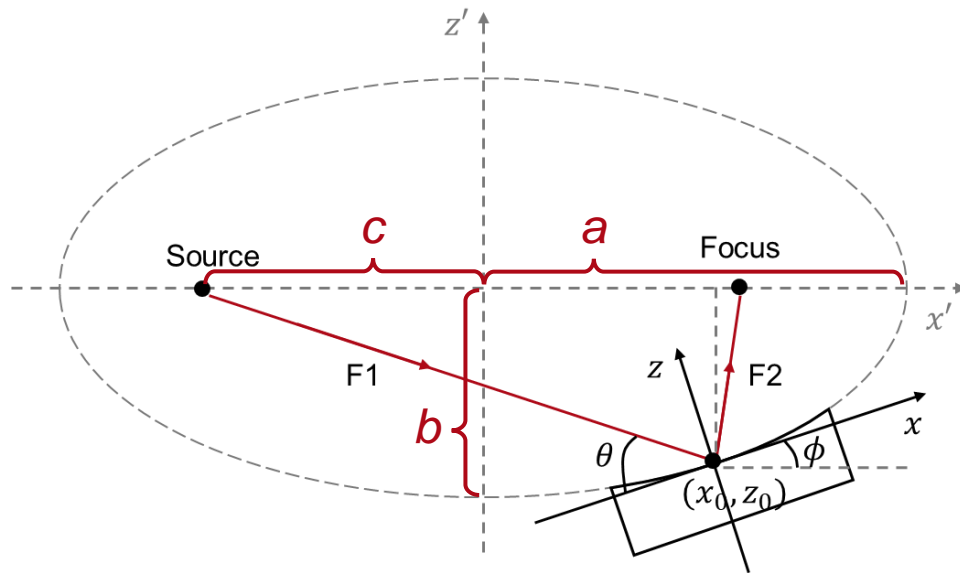
1<sup>st</sup> elliptical mirror

- $F_1 = 50\text{m}$
- $F_2 = 400\text{mm}$
- $\theta = 0.172^\circ$

2nd elliptical mirror

- $F_1 = 50.2\text{m}$
- $F_2 = 200\text{mm}$
- $\theta = 0.172^\circ$

# Analytical Design of the Elliptical Mirror (2)



- To calculate the height function  $z(x)$ , two equations need to be considered.

- elliptical equation

$$\frac{x'^2}{a^2} + \frac{z'^2}{b^2} = 1$$

- coordinate transform

$$\begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} x_0 \\ z_0 \end{pmatrix}$$

- The final height function  $z(x)$  is

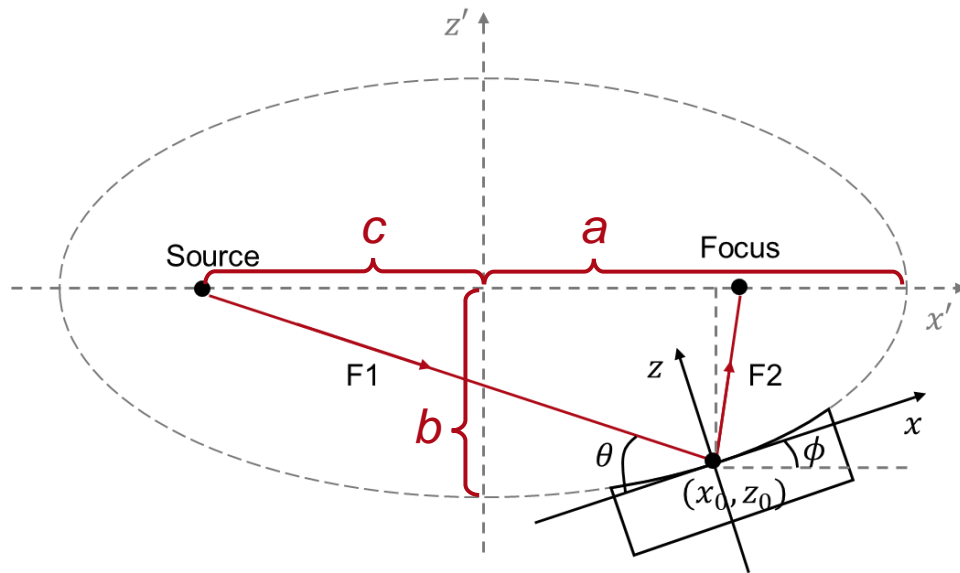
$$z(x) = (z' - z_0) \cos \phi - (x' - x_0) \sin \phi$$

with  $\phi = \arctan \left( -\frac{b^2}{a^2} \frac{x_0}{z_0} \right)$ ,  $x_0 = \frac{F_1^2 - F_2^2}{4c}$ , and

$$z_0 = -b * \sqrt{1 - x_0^2/a^2}$$

unknowns  $a, b, x'$  and  $z'$  in next slide

# Analytical Design of the Elliptical Mirror (3)



- $a$  and  $b$  can be calculated from  $F_1$ ,  $F_2$  and  $\theta$

$$a = \frac{F_1 + F_2}{2}$$

$$c = \frac{1}{2} \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos(\pi - 2\theta)}$$

$$b = \sqrt{a^2 - c^2}$$

- parameter  $z'$  and  $x'$  are

$$z'(x') = -b \sqrt{1 - \frac{x'^2}{a^2}}, \quad x'(x) = \frac{-n + \sqrt{n^2 - 4mt}}{2m}$$

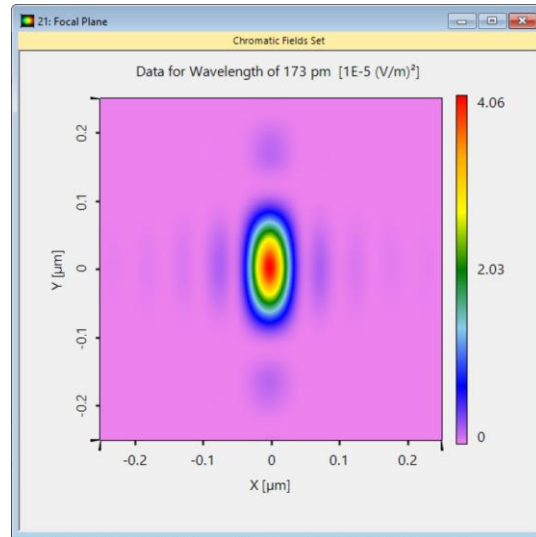
with

$$m = \cos^2 \phi + \frac{b^2}{a^2} \sin^2 \phi$$

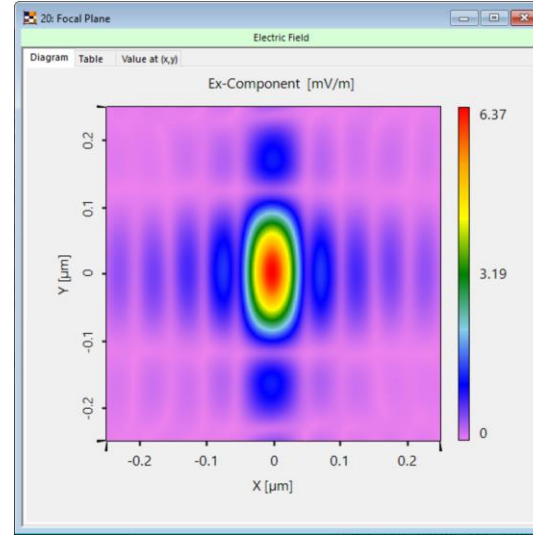
$$n = -2 \cos \phi (x + x_0 \cos \phi + z_0 \sin \phi)$$

$$t = (x + x_0 \cos \phi + z_0 \sin \phi)^2 - b^2 \sin^2 \phi$$

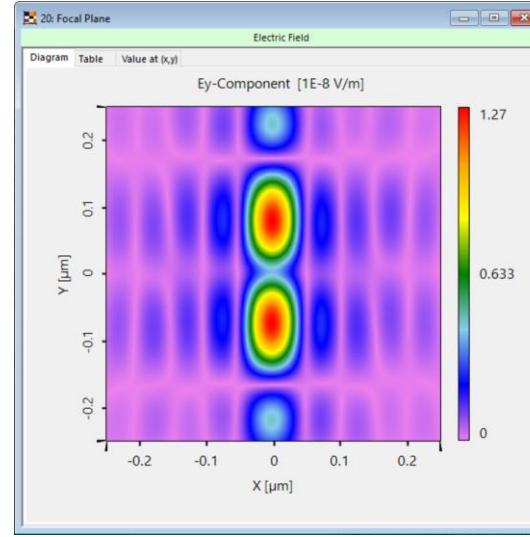
# Energy Density & E-Field at Focal Plane



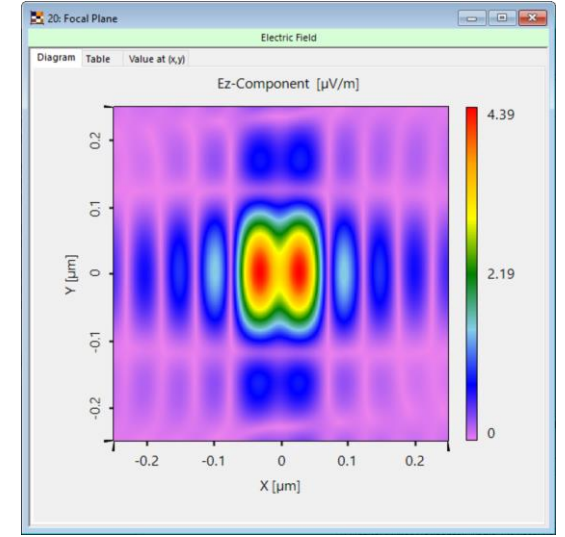
energy density



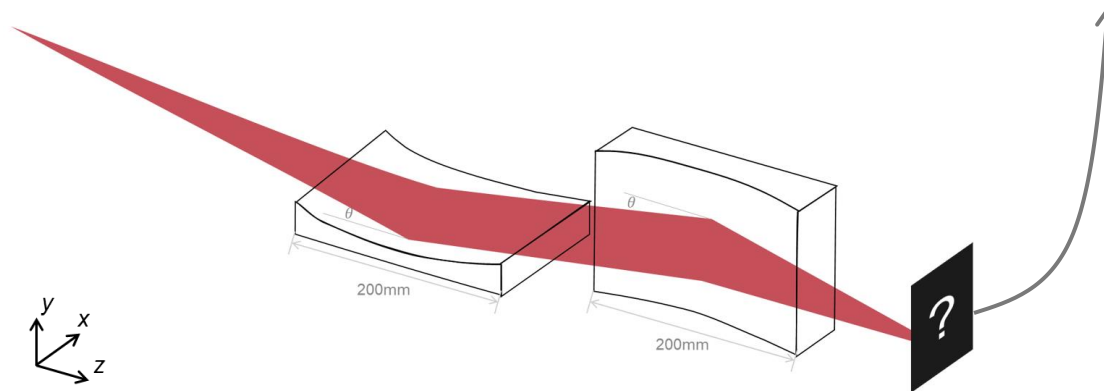
amplitude of  $E_x$



amplitude of  $E_y$



amplitude of  $E_z$



- The vectorial field in the focus is calculated.
- The size of the diffraction pattern is at nanometer-scale.



# Peek into VirtualLab Fusion

**Edit Programmable Surface**



Structure | Height Discontinuities | Scaling | Coating | Periodization

Surface Specification

Algorithms

Snippet for Height Profile  Validity: 

☐ Numerical Gradient Calculation

☒ User-Defined Gradient Calculation  Validity: 

Parameters

Angle

F1

F2

Source Code Editor

Source Code | Global Parameters | Snippet Help | Advanced Settings


```
50 ***** INSERT YOUR CODE HERE *****
51 *****
52
53 double a = (F1 + F2) / 2; // distance from center to
54 double c = Math.Sqrt(F1 * F1 + F2 * F2 - 2 * F1 * F2
55 double b = Math.Sqrt(a * a - c * c); // distance from
56
57 double x0 = (F1 * F1 - F2 * F2) / (4 * c); // (x0, z0
58 double z0 = -b * Math.Sqrt(1 - x0 * x0 / (a * a));
59
60 // now is to shift and rotate the coordinate the mir
61 double phi = Math.Atan(-b * b * x0 / (a * a * z0));
62 double sin = Math.Sin(phi);
63 double cos = Math.Cos(phi);
64
65 double m = b * b * sin * sin / (a * a) + cos * cos;
66 double n = -2 * cos * (x + x0 * cos + z0 * sin);
67 double t = (x + x0 * cos + z0 * sin) * (x + x0 * cos
68 double x1 = (-n + Math.Sqrt(n * n - 4 * m * t)) / (2
69 double z1 = -b * Math.Sqrt(1 - x1 * x1 / (a * a));
70
71 height = -1 * (-x1 - x0) * sin + (z1 - z0) * cos; //
```


flexible definition of customized interface

**Edit Curved Surface Component**

Basal Positioning | Isolated Positioning | Position Information (Absolute)

Position this Element's Input Axes with Respect to

Reference Element   Enter Absolute Positioning Data

Reference Output Coordinate System  



Relative Distance on Axis

Delta Z



Lateral Shift



Delta X  Delta Y

Inclination / Rotation

Orientation Definition Type   

Z-Axis Direction Definition

Angle / Axis	Value
Theta (Spherical) 	89.828°
Phi (Spherical) 	-90°

Swap Order  

Rotation About Z-Axis

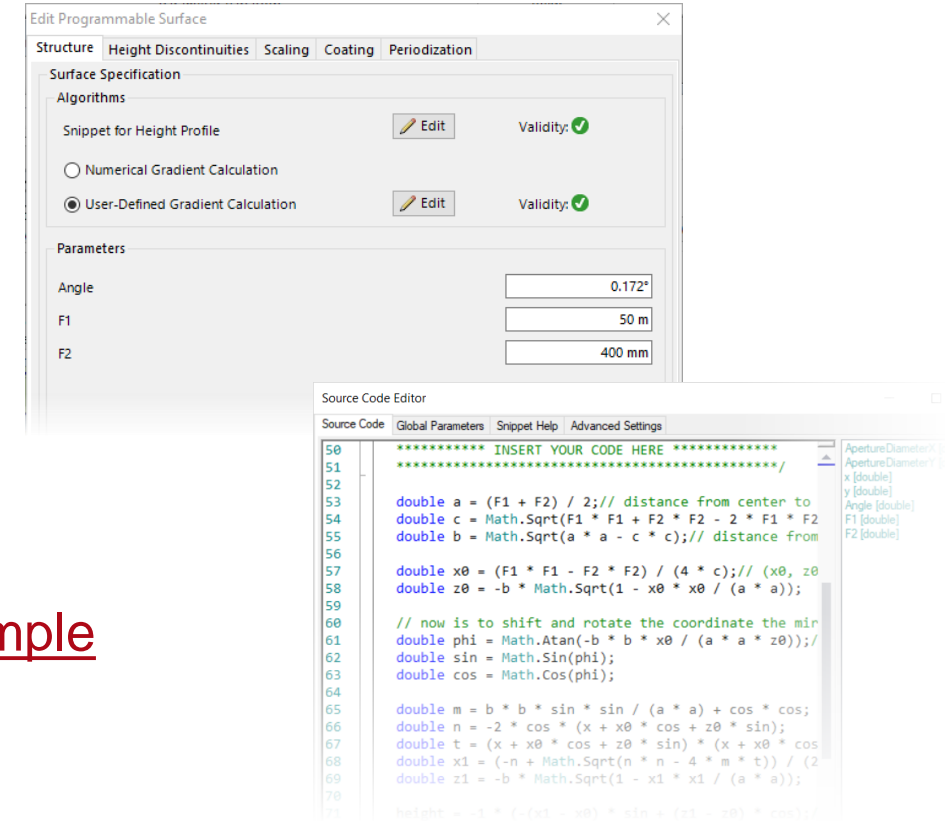
Z-Axis Rotation Angle

convenient definition of position and orientation

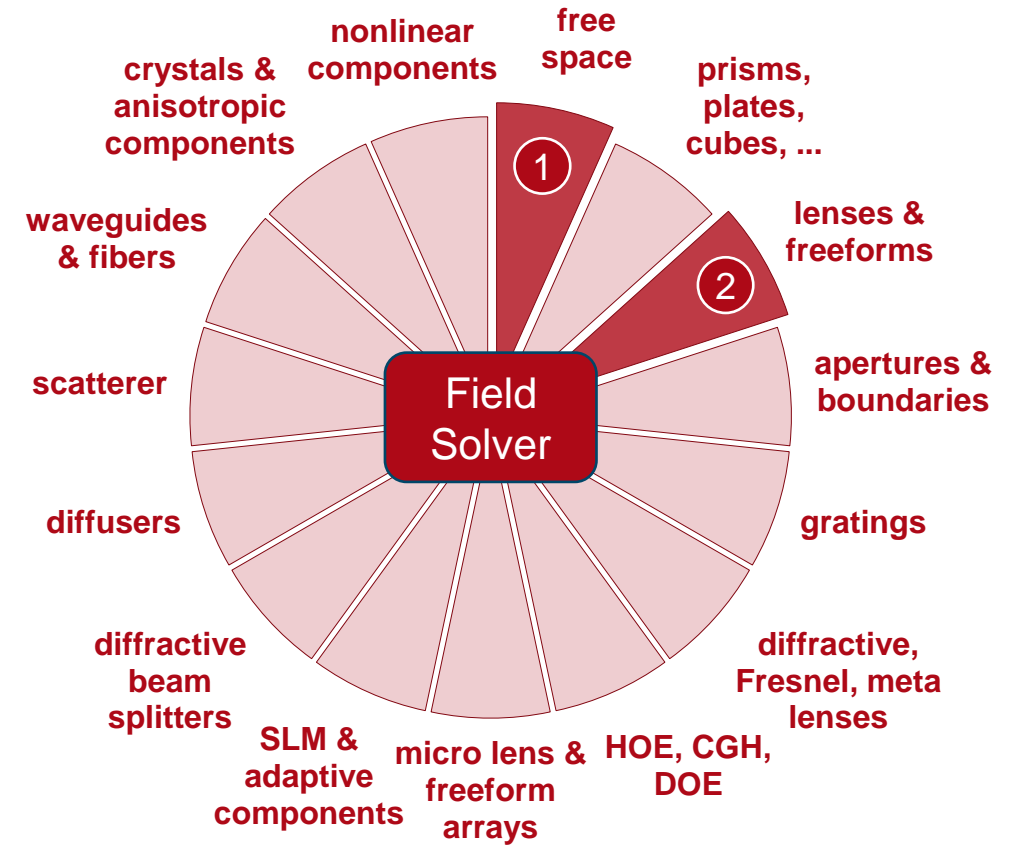
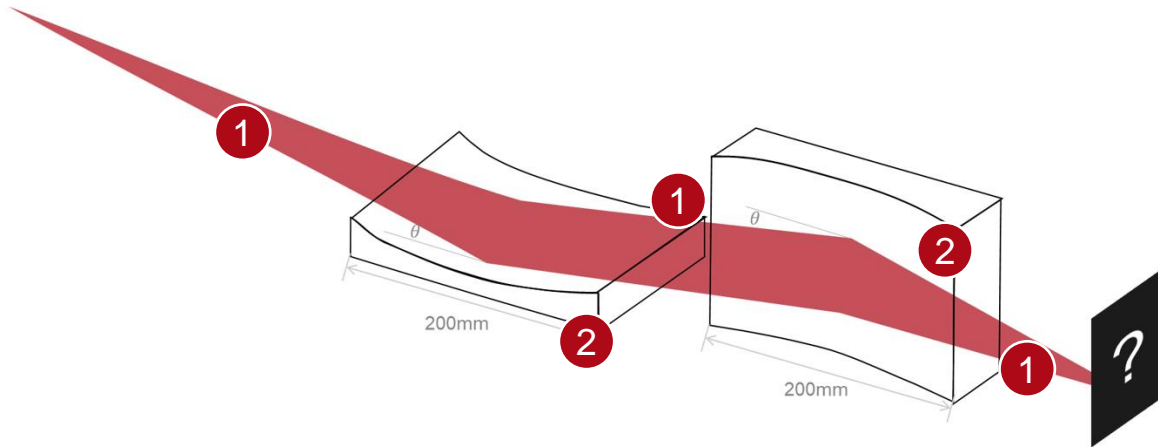


# Workflow in VirtualLab Fusion

- Set up input Gaussian field
  - [Basic Source Models](#) [Tutorial Video]
- Set the position and orientation of components
  - [LPD II: Position and Orientation](#) [Tutorial Video]
- Programmable the elliptical interface
  - [How to Work with the Programmable Interface & Example \(Spherical Surface\)](#) [Use Case]



# VirtualLab Fusion Technologies



# idealized component

# Document Information

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title	Grazing-Incidence Focusing Mirrors for X-Ray Beams
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software edition	VirtualLab Fusion Basic
software version	2021.1 (Build 1.180)
category	Application Use Case
further reading	- <a href="#"><u>Single Grating Interferometer for X-Ray Imaging</u></a>