VirtualLab Fusion Technology – Solvers and Functions

# Idealized Lens Functions

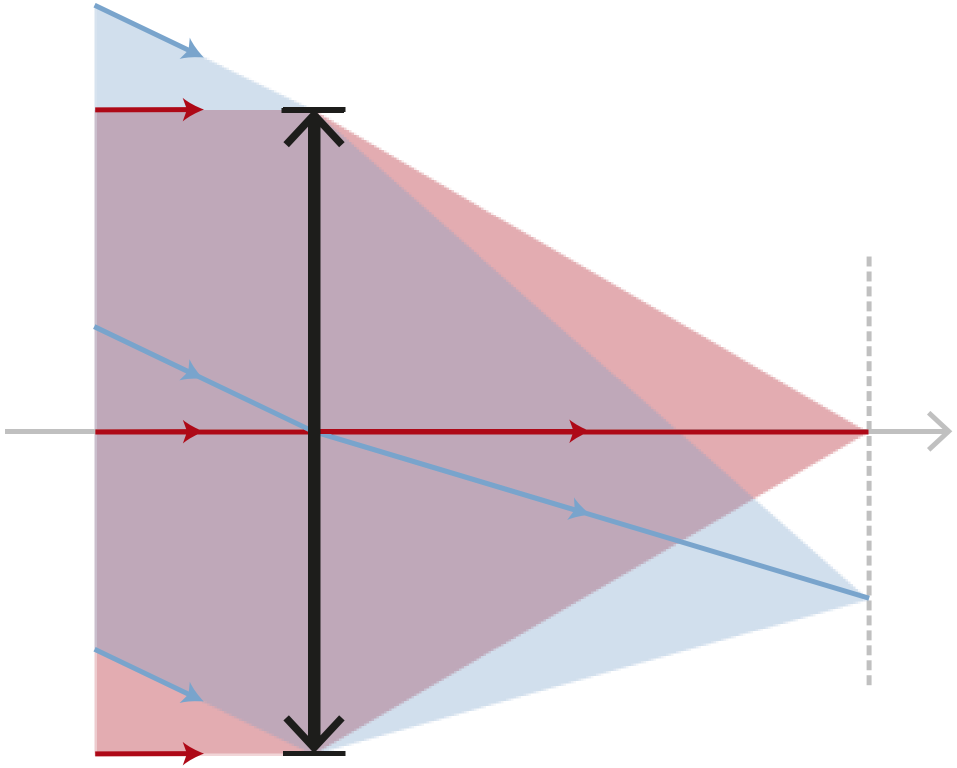
For the **Idealized Lens Component** (from Component Catalog)

# Abstract


The idealized lens function works without information about the actual lens surface and material. In the spatial (x) domain, it defines

1. the change of wavefront phase according to different lens types, and, therefore
2. also the change of the spatial frequencies (ray directions) that follows from the wavefront phase.

The lens functions are defined in focusing and collimation mode respectively, as their design working modes. Additionally, three different lens function types are available – they defines the relation between e.g. the angle of incidence and the location of the focal position.

# Function Algorithm

- Both the input and output fields are defined on the plane reference surface, in the following form

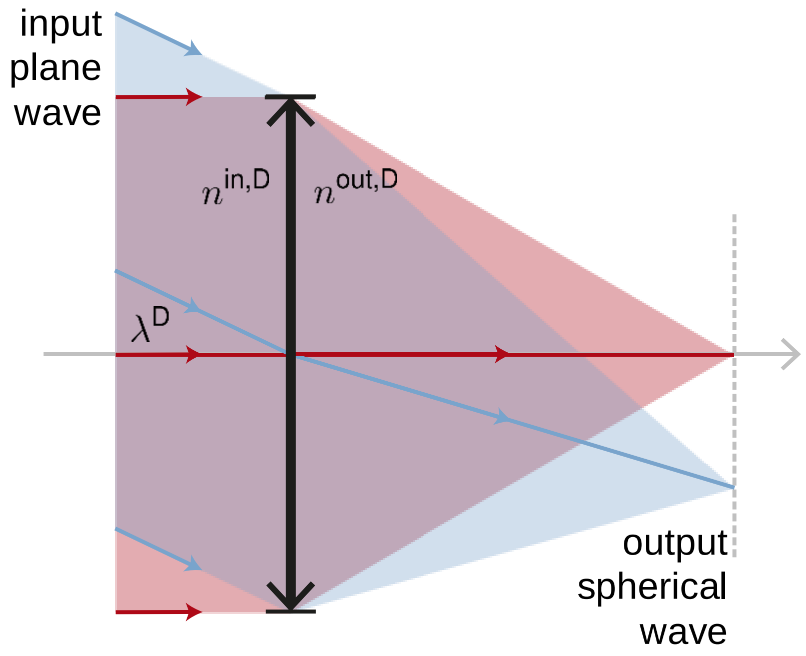
$$\boldsymbol{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}, z) = \boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}, z) \exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho}, z)\right),$$
$$\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}, z) = \boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}, z) \exp\left(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho}, z)\right),$$

  where $\psi$ is the wavefront phase part, $\boldsymbol{U}_{\perp}$ is the residual fields, and $\boldsymbol{\rho} = (x, y)$ as the transverse coordinates.

- The idealized lens functions are defined for the **design wavelength $\lambda^{\text{D}}$**, **design refractive indices $n^{\text{in,D}}$ and $n^{\text{out,D}}$**, and the output field is to be calculated pointwisely

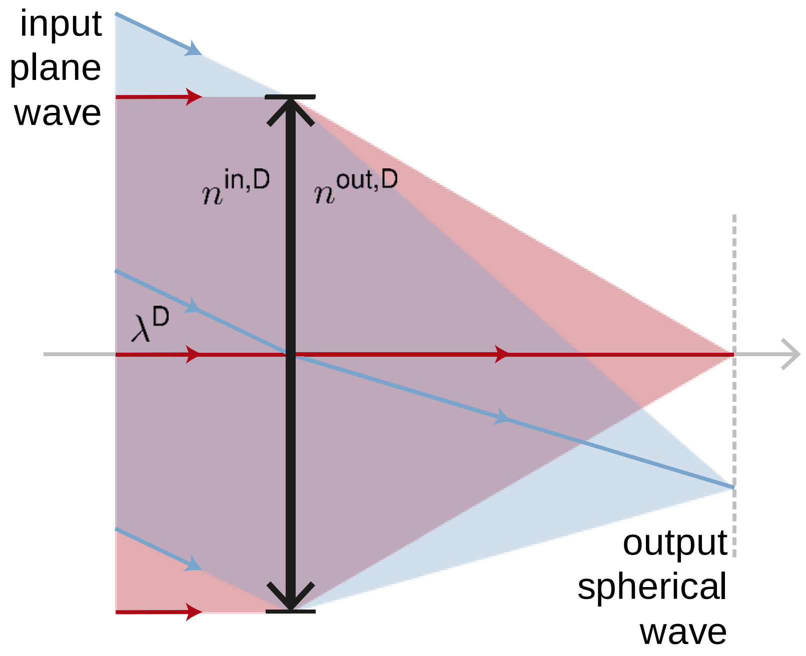
$$\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}, z) = \mathbf{B}(\boldsymbol{\rho}, z)\boldsymbol{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}, z),$$

  or, explicitly, with
  - the redisual field: $\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}, z) = \mathbf{b}(\boldsymbol{\rho}, z)\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}, z)$, and
  - the wavefront phase part: $\psi^{\text{out}}(\boldsymbol{\rho}, z) = \psi^{\text{in}}(\boldsymbol{\rho}, z)$.



# Function Algorithm

- To summarize it, the lens function can be expressed as
  - $\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})$;
  - $\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \Delta\boldsymbol{\kappa}(\boldsymbol{\rho})$;
  - $\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{b}(\boldsymbol{\rho})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$.
- And to explain the above
  - the change in the wavefront phase $\Delta\psi(\boldsymbol{\rho})$ corresponds to the major lens function, and it will be determined according to different **lens types**;
  - the change in spatial frequency $\Delta\boldsymbol{\kappa}(\boldsymbol{\rho})$ follows from $\Delta\psi(\boldsymbol{\rho})$ completely and it is directly related to the bending of ray direction;
  - the change in the field vector components $\mathbf{b}(\boldsymbol{\rho})$ follows the rule of idealized grating functions, as a special case.

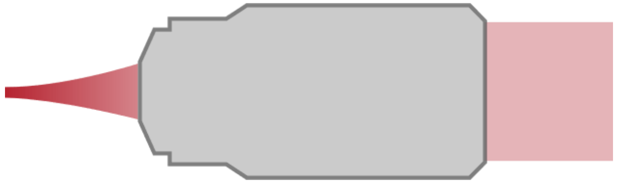


# Lens Modes Definition

- The lens function can be considered, with its design parameters and in its design situation, as follows:
  1. It focuses an input plane wavefront into spherical one, with the focal location dependent on input direction (**focusing mode**);
  2. Reversely, it collimates an input spherical wavefront (from a point on the focal plane) into a plane one, with the output direction dependent on the input point location (**collimation mode**).
- **For focusing mode**, let us consider plane waves that covers the whole lens area in x domain, scanning with different angles, and the full response at any $\boldsymbol{\rho}$ for any $\boldsymbol{\kappa}^{\text{in}}$ can be found.
- **For collimation mode**, we consider point sources located at different lateral positions, which emit light cones converning the whole lens area, and the full response at any $\boldsymbol{\rho}$ for any $\boldsymbol{\kappa}^{\text{in}}$ can be found.

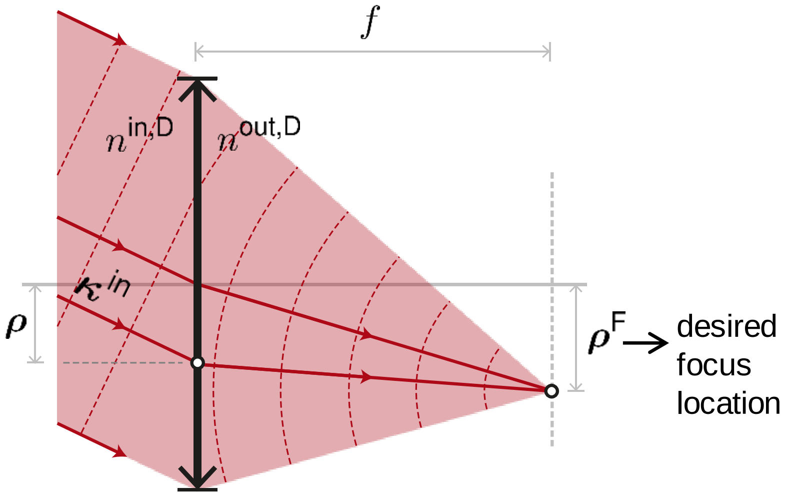
focusing mode

collimation mode

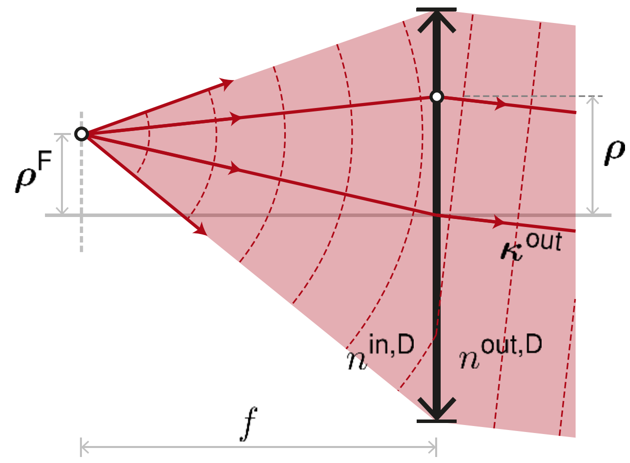


# Lens Functions in Different Modes

- **For focusing mode**, we can find that
  - The input wavefront phase $\psi^{\text{in}} = \boldsymbol{\kappa}^{\text{in}} \cdot \boldsymbol{\rho}$;
  - The output wavefront phase $\psi^{\text{out}} = -k_0^{\text{D}} n^{\text{out,D}} r$, with $r = \sqrt{||\boldsymbol{\rho} - \boldsymbol{\rho}^{\text{F}}||^2 + f^2}$;
  - In this case, $\boldsymbol{\rho}^{\text{F}}$ must be calculated according to a particular **lens mode** with respect to $\boldsymbol{\kappa}^{\text{in}}$, and then $r$ and $\boldsymbol{\kappa}^{\text{out}}$ will follow.



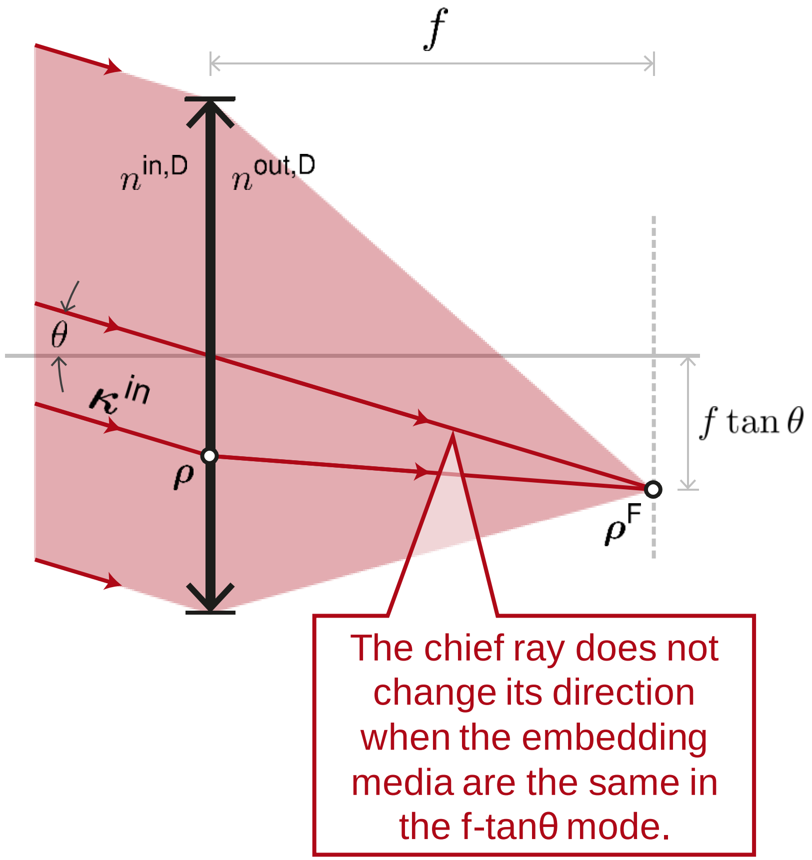

- **For collimation mode**, in a reversed manner, we can find
  - The input wavefront phase $\psi^{\text{in}} = k_0^{\text{D}} n^{\text{in,D}} r$, with $r = \sqrt{||\boldsymbol{\rho} - \boldsymbol{\rho}^{\text{F}}||^2 + f^2}$;
  - The output wavefront phase $\psi^{\text{out}} = \boldsymbol{\kappa}^{\text{out}} \cdot \boldsymbol{\rho}$;
  - In this case, $\boldsymbol{\rho}^{\text{F}}$ is fixed (and thus for $r$ too) for a given $\boldsymbol{\rho}$ and $\boldsymbol{\kappa}^{\text{in}}$, and $\boldsymbol{\kappa}^{\text{out}}$ must be calculated according to a particular **lens mode**.



# Lens Function Types (Focusing Mode)

- **For the IO configuration**: the question is how to calculate $\boldsymbol{\rho}^{\mathsf{F}}$ from $\boldsymbol{\kappa}^{\mathsf{in}}$?
- We list the following three lens modes
  - $f$-$\tan\theta$ mode: $\boldsymbol{\rho}^{\mathsf{F}} = f\tan\theta\frac{\boldsymbol{\kappa}^{\mathsf{in}}}{||\boldsymbol{\kappa}^{\mathsf{in}}||} = f\frac{\boldsymbol{\kappa}^{\mathsf{in}}}{k_z^{\mathsf{in,D}}}$, with $k_z^{\mathsf{in,D}} = \sqrt{(k_0^{\mathsf{D}}n^{\mathsf{in,D}})^2 - ||\boldsymbol{\kappa}^{\mathsf{in}}||^2}$;
  - $f$-$\sin\theta$ mode: $\boldsymbol{\rho}^{\mathsf{F}} = f\sin\theta\frac{\boldsymbol{\kappa}^{\mathsf{in}}}{||\boldsymbol{\kappa}^{\mathsf{in}}||} = f\frac{\boldsymbol{\kappa}^{\mathsf{in}}}{k_0^{\mathsf{D}}n^{\mathsf{in,D}}}$;
  - $f$-$\theta$ mode: $\boldsymbol{\rho}^{\mathsf{F}} = f\theta\frac{\boldsymbol{\kappa}^{\mathsf{in}}}{||\boldsymbol{\kappa}^{\mathsf{in}}||}$, with $\theta = \sin^{-1}\left[\frac{||\boldsymbol{\kappa}^{\mathsf{in}}||}{k_0^{\mathsf{D}}n^{\mathsf{in,D}}}\right]$.
- Above, the term $\boldsymbol{\kappa}^{\mathsf{in}}/||\boldsymbol{\kappa}^{\mathsf{in}}||$ can be regarded as a normalized direction vector. By substituting the expression of $\sin\theta$ and $\tan\theta$, the corresponding simplified forms are obtained as above.
- With $\boldsymbol{\rho}^{\mathsf{F}}$ determined, we can fix $\boldsymbol{\kappa}^{\mathsf{out}} = k_0^{\mathsf{D}}n^{\mathsf{out,D}}(\boldsymbol{\rho}^{\mathsf{F}} - \boldsymbol{\rho})/r$.

illustration according to the f-tanθ mode



# Lens Function Types (Collimation Mode)

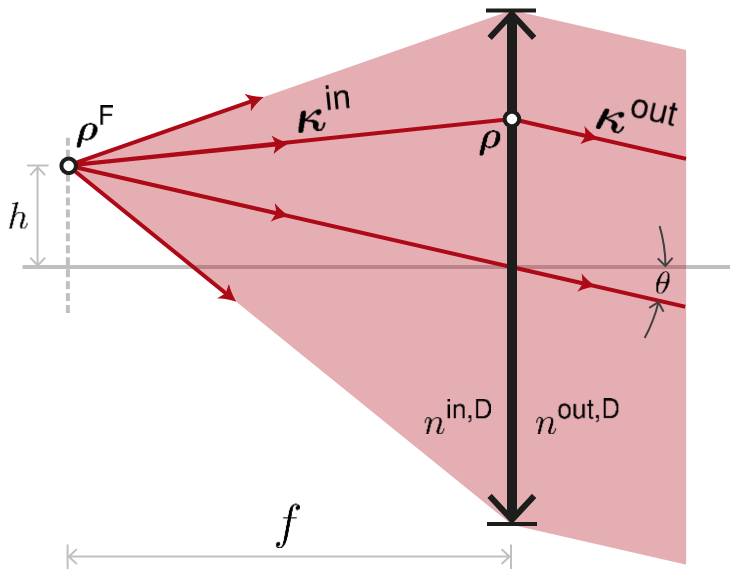
- **For the OI configuration**: the focal location is fixed as

$$\boldsymbol{\rho}^{\mathsf{F}} = \boldsymbol{\rho} - f\boldsymbol{\kappa}^{\mathsf{in}}/k_z^{\mathsf{in,D}} = \boldsymbol{\rho} - f\hat{\boldsymbol{s}}_{\perp}^{\mathsf{in}}/\hat{s}_z^{\mathsf{in}}\,,$$

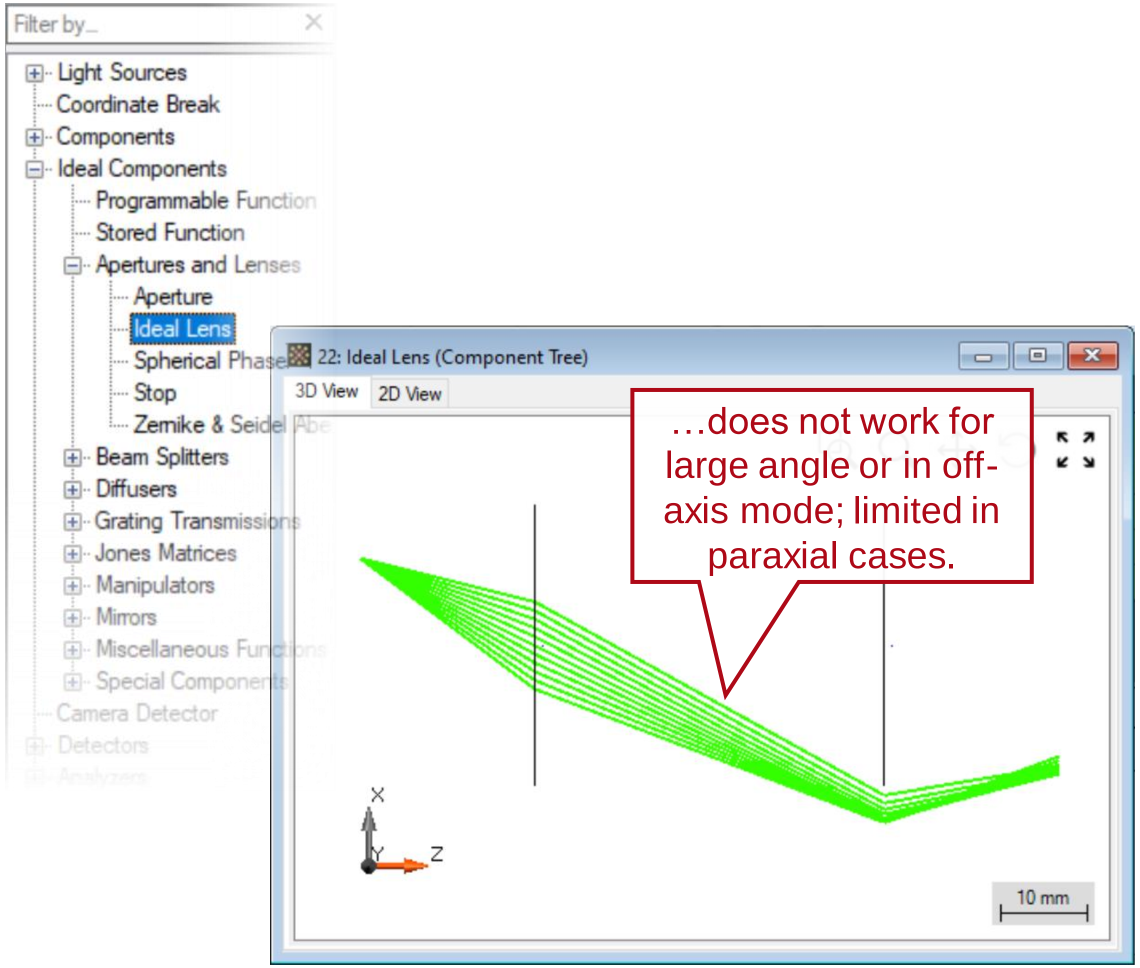
  for any $\boldsymbol{\rho}$ and $\boldsymbol{\kappa}^{\mathsf{in}}$, and the question is how to determine $\boldsymbol{\kappa}^{\mathsf{out}}$?

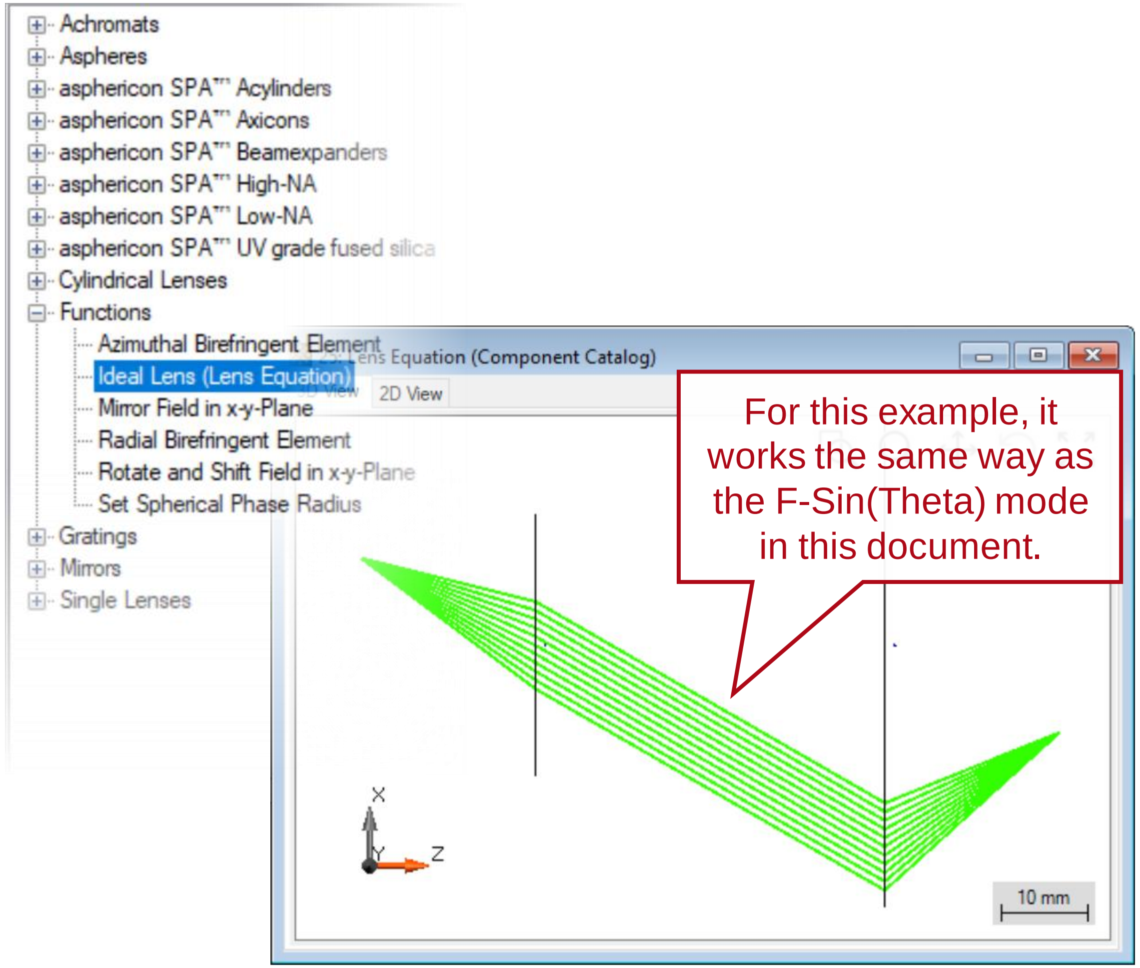
- We find reverted relations for the following three lens modes
  - $f$-$\tan\theta$ mode: $\boldsymbol{\kappa}^{\mathsf{out}} = -k_z^{\mathsf{out,D}}\frac{\boldsymbol{\rho}^{\mathsf{F}}}{f}$,
    with $k_z^{\mathsf{out,D}} = \left[(k_0^{\mathsf{D}}n^{\mathsf{in,D}})^2/(1+||\boldsymbol{\rho}^{\mathsf{F}}/f||^2)\right]^{-\frac{1}{2}}$;
  - $f$-$\sin\theta$ mode: $\boldsymbol{\kappa}^{\mathsf{out}} = -k_0^{\mathsf{D}}n^{\mathsf{out,D}}\frac{\boldsymbol{\rho}^{\mathsf{F}}}{f}$;
  - $f$-$\theta$ mode: $\boldsymbol{\kappa}^{\mathsf{out}} = -||\boldsymbol{\kappa}^{\mathsf{out}}||\frac{\boldsymbol{\rho}^{\mathsf{F}}}{||\boldsymbol{\rho}^{\mathsf{F}}||}$,
    with $||\boldsymbol{\kappa}^{\mathsf{out}}|| = k_0^{\mathsf{D}}n^{\mathsf{out,D}}\sin\theta$, and $\theta = ||\boldsymbol{\rho}^{\mathsf{F}}||/f$.

illustration according to the f-tanθ mode



# Comparison with Other Models (Example #1)

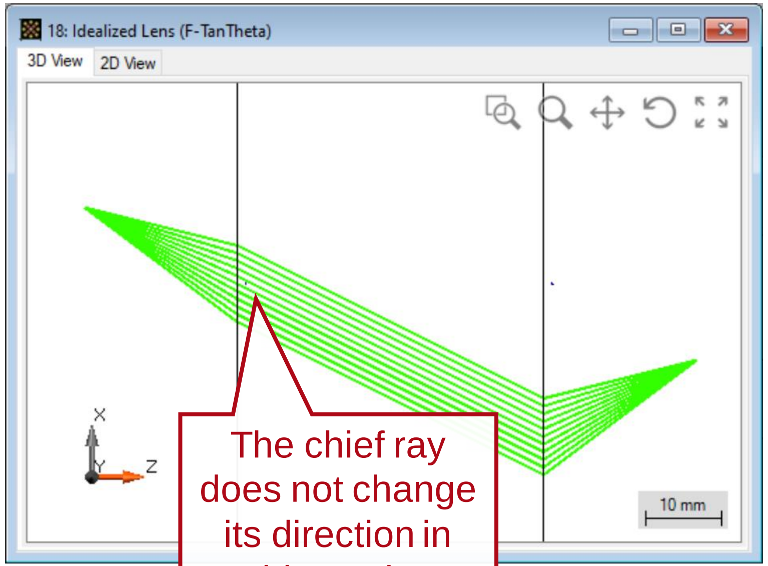


# Comparison with Other Models (Example #1)

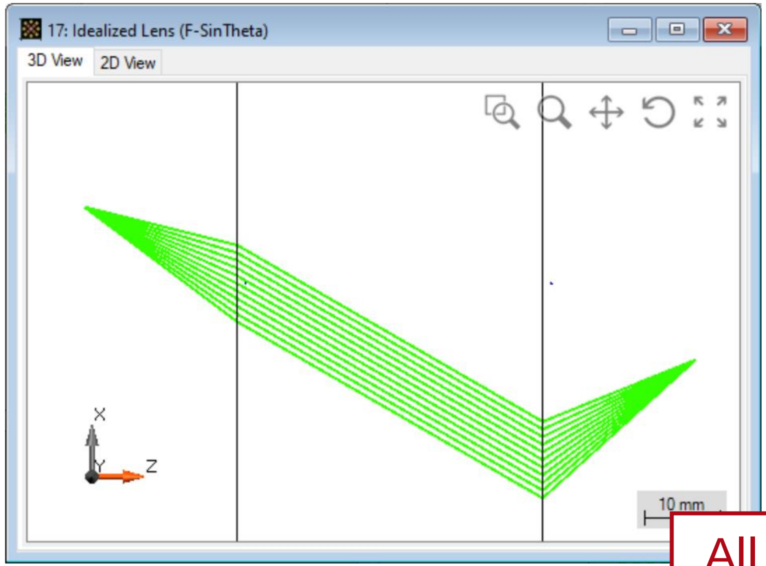
idealized lens functions according to this document…

(a) $f$-$\tan\theta$ mode (default)

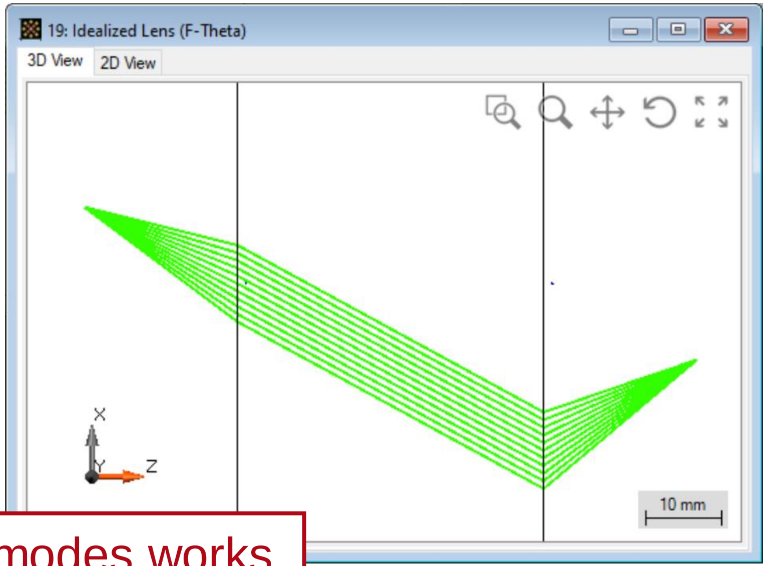



The chief ray does not change its direction in this mode.

(b) $f$-$\sin\theta$ mode

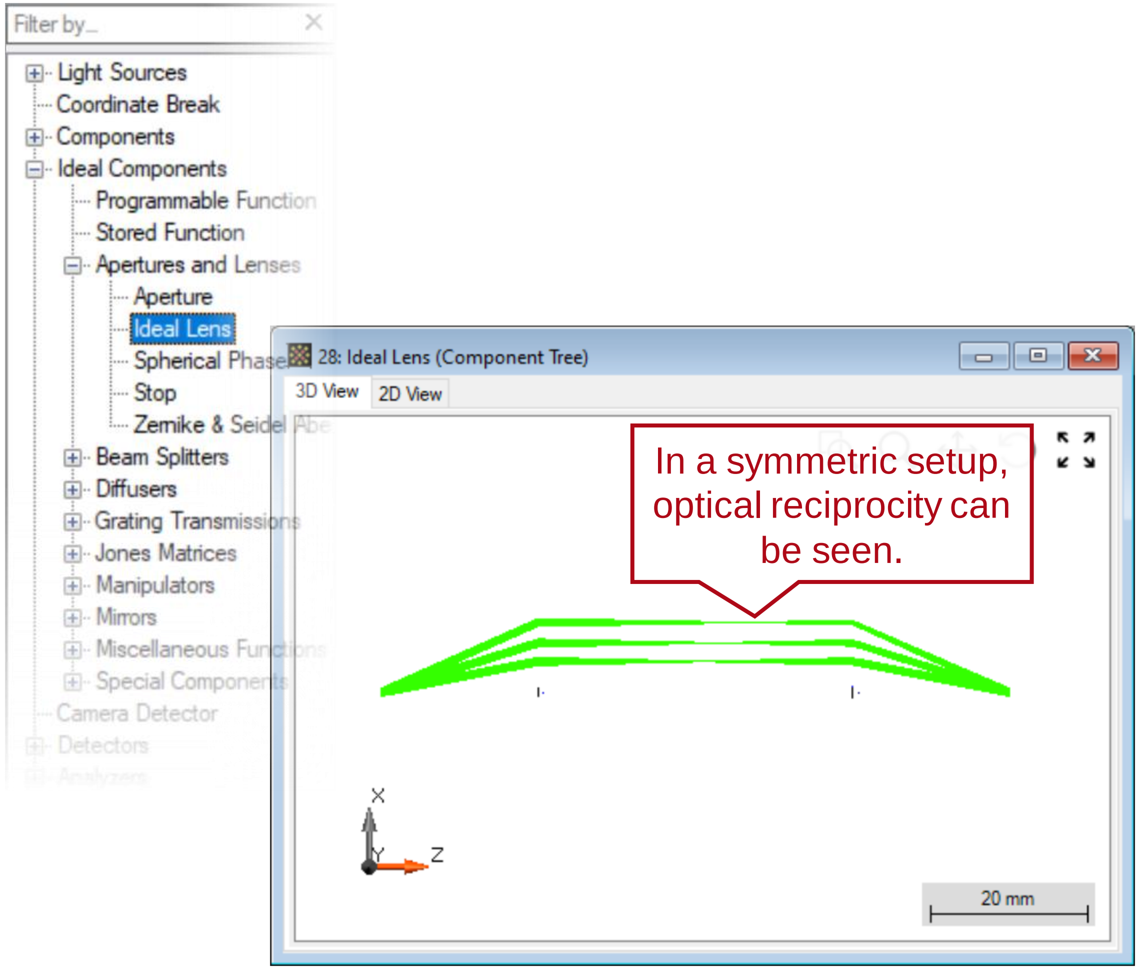


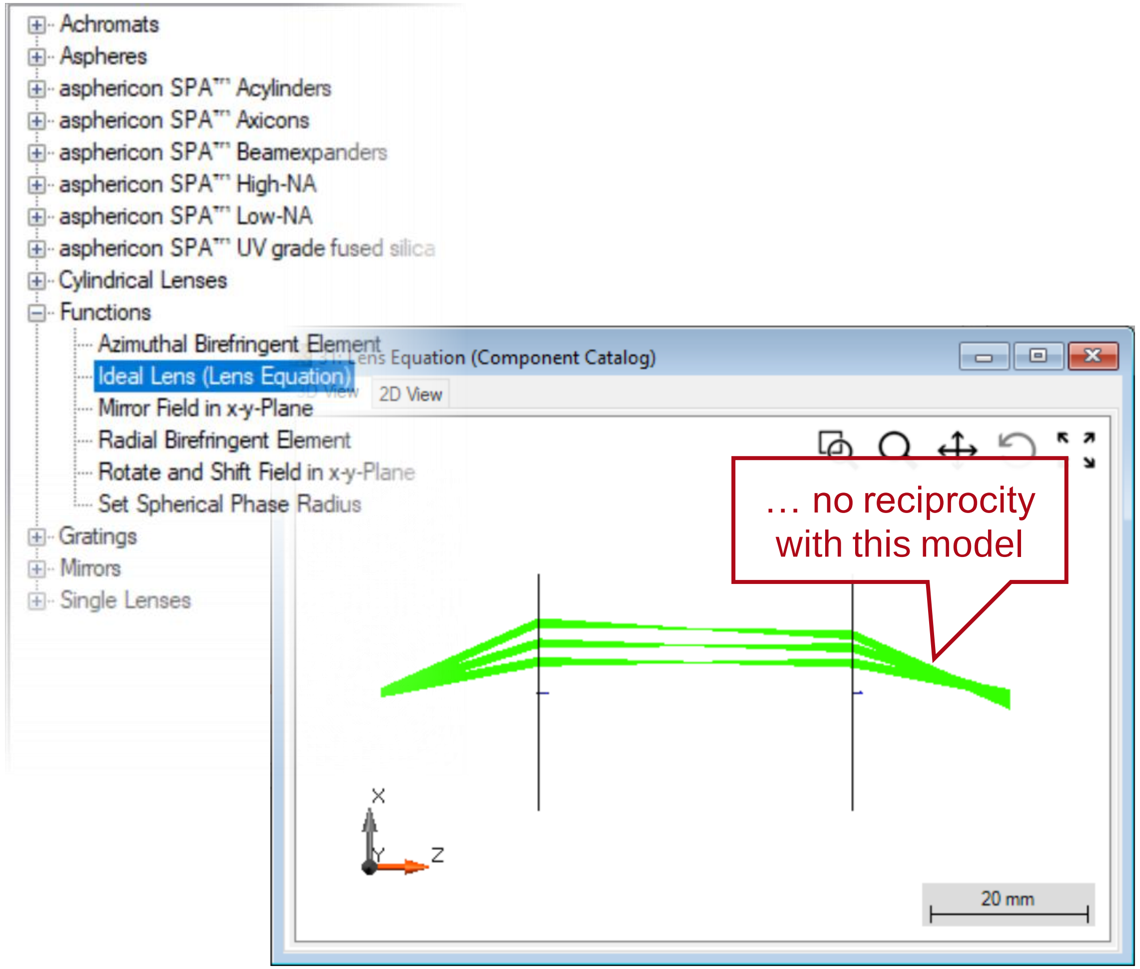

(c) $f$-$\theta$ mode




All modes works beyond the paraxial cases.

# Comparison with Other Models (Example #2)

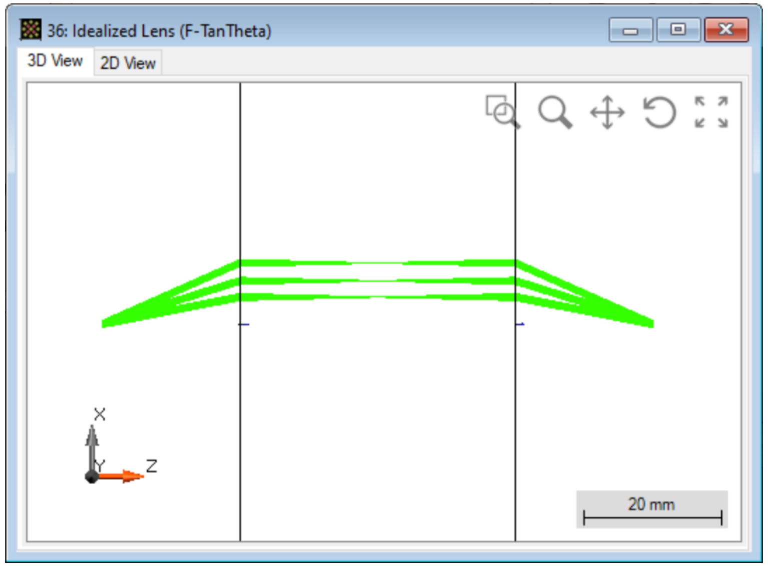


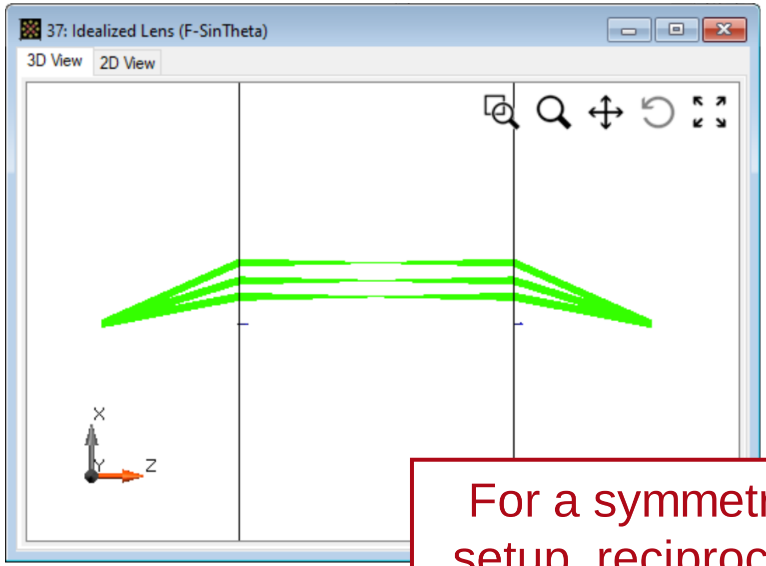
# Comparison with Other Models (Example #2)

idealized lens functions according to this document…

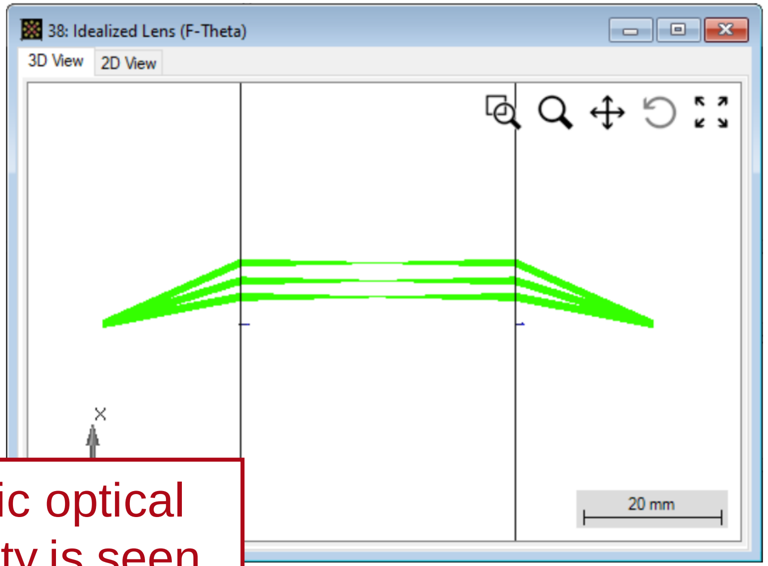
(a) $f\text{-}\tan\theta$ mode (default)




(b) $f\text{-}\sin\theta$ mode




(c) $f\text{-}\theta$ mode




For a symmetric optical setup, reciprocity is seen in all three lens modes.

# Document Information

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