VirtualLab Fusion Technology – Solvers and Functions

Idealized Grating Functions

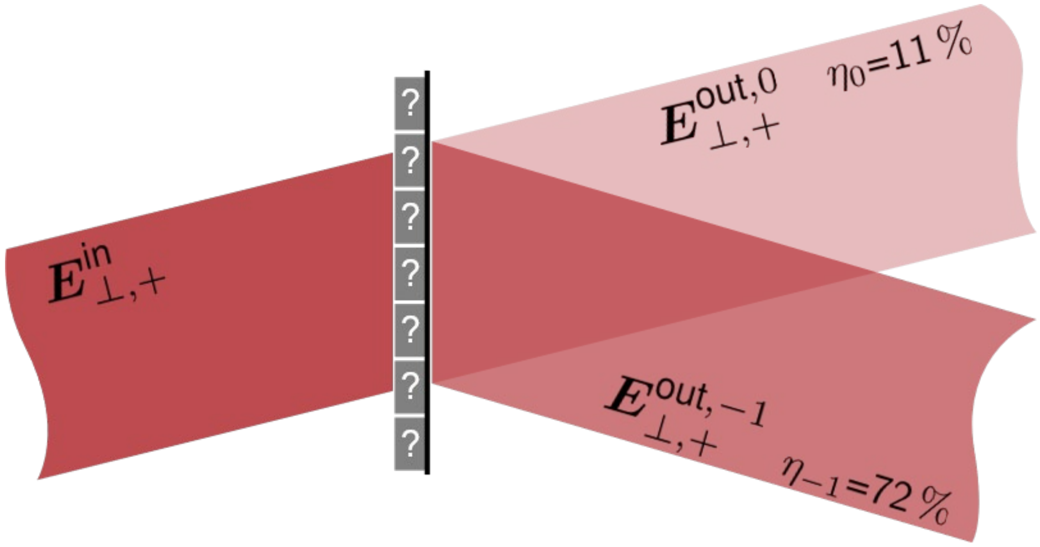
For the **Functional Grating Component**

Abstract

The idealized grating function works without information about the actual shape of the grating structure. In the spatial frequency (k) domain, it defines

1. the position (in k domain) of the diffraction orders according to the grating equation, and
2. the effective B-matrix for each order according to a desired diffraction efficiency.

The relationship between the position of an arbitrary diffraction order and that of the input is governed by the grating equation. For a specific order, to connect the outgoing electromagnetic field quantities with the incoming ones, a 2×2 B-matrix is needed. An idealized method is used to define such an effective B-matrix from a given diffraction efficiency.



Function Algorithm – Grating Equation

- In the k domain, the grating equation can be expressed as

$$\boldsymbol{\kappa}^{\text{out}} = \boldsymbol{\kappa}^{\text{in}} + j_x \frac{2\pi}{d_x} + j_y \frac{2\pi}{d_y} \,,$$

  where

  - $\boldsymbol{\kappa}^{\text{out}}$ is the transverse spatial frequency of an arbitrary diffraction order, with **order indices** j_x and j_y along x and y respectively;
  - $\boldsymbol{\kappa}^{\text{in}}$ is the input transverse spatial frequency;
  - d_x and d_y are the **grating periods** along x and y respectively.

- From the transverse spatial frequencies, the corresponding diffraction angle can be calculated if needed. In this document we keep working in the k domain and only leave it when required.

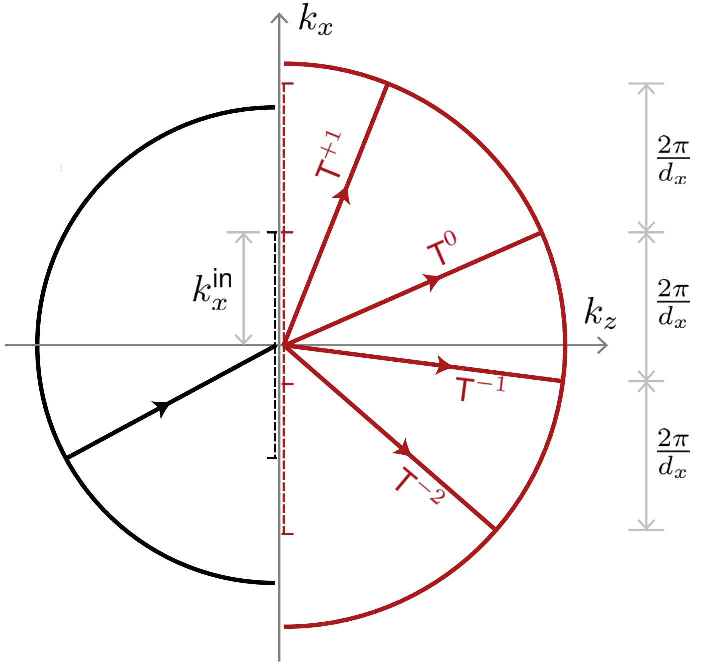


illustration of the grating equation in the k domain for a 1D grating along the x direction

Function Algorithm – From B-Matrix to Efficiency

- Let us consider the following input plane wave

$$\boldsymbol{E}_{\perp}^{\text{in}}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}_{\perp}^{\text{in}} \exp(\mathrm{i}\boldsymbol{\kappa}^{\text{in}} \cdot \boldsymbol{\rho}) \exp(\mathrm{i}k_z^{\text{in}} z)\,,$$

with given $\boldsymbol{\kappa}^{\text{in}}$ and $\tilde{\boldsymbol{E}}_{\perp}^{\text{in}} = (\tilde{E}_x^{\text{in}}, \tilde{E}_y^{\text{in}})$.

- For a specific diffraction order, the output plane wave can be expressed as

$$\boldsymbol{E}_{\perp}^{\text{out}}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}_{\perp}^{\text{out}} \exp(\mathrm{i}\boldsymbol{\kappa}^{\text{out}} \cdot \boldsymbol{\rho}) \exp(\mathrm{i}k_z^{\text{out}} z)\,,$$

with $\boldsymbol{\kappa}^{\text{out}}$ determined via the grating equation, and the transverse field components connected by a 2×2 B-matrix:

$$\begin{pmatrix} \tilde{E}_x^{\text{out}} \\ \tilde{E}_y^{\text{out}} \end{pmatrix} = \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \tilde{E}_x^{\text{in}} \\ \tilde{E}_y^{\text{in}} \end{pmatrix}.$$

- For a homogeneous isotropic medium with constant relative permittivity ϵ and permeability μ, we can calculate

$$k_z = \sqrt{k_0^2 \epsilon \mu - k_x^2 - k_y^2}\,,$$

and

$$\tilde{E}_z = -\frac{k_x \tilde{E}_x + k_y \tilde{E}_y}{k_z}\,.$$

Function Algorithm – From B-Matrix to Efficiency

- We restrict ourselves to the case of **lossless** media (real-valued ϵ and μ), and in such cases, the time-averaged Poynting vector can be expressed as

$$\boldsymbol{S} = \left\langle \bar{\boldsymbol{S}}^{(\mathrm{r})}(t) \right\rangle = \frac{1}{2\omega\mu_0\mu} ||\tilde{\boldsymbol{E}}||^2 \boldsymbol{k} \,,$$

  with its direction coinciding with that of the wavevector $\boldsymbol{k}$.

- Applying the relation above, the **diffraction efficiency** of a specific order can be calculated (with respect to the input) via

$$\eta = \frac{S_z^{\mathrm{out}}}{S_z^{\mathrm{in}}} = \frac{\mu^{\mathrm{in}} ||\tilde{\boldsymbol{E}}^{\mathrm{out}}||^2 k_z^{\mathrm{out}}}{\mu^{\mathrm{out}} ||\tilde{\boldsymbol{E}}^{\mathrm{in}}||^2 k_z^{\mathrm{in}}} \,,$$

  where S_z and k_z are the z components of the time-averaged Poynting vector and the wavevector respectively.

- Here $\tilde{\boldsymbol{E}}$ is a three-dimensional vector:

$$\tilde{\boldsymbol{E}} = \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \end{pmatrix} .$$

Function Algorithm – From Efficiency to B-Matrix

- From the diffraction efficiency η, we can only conclude the following relation
$$\frac{||\tilde{\boldsymbol{E}}^{\text{out}}||^2}{||\tilde{\boldsymbol{E}}^{\text{in}}||^2} = \frac{\mu^{\text{out}} k_z^{\text{in}}}{\mu^{\text{in}} k_z^{\text{out}}} \eta \, .$$

- Obviously, the ratio between $||\tilde{\boldsymbol{E}}^{\text{out}}||^2$ and $||\tilde{\boldsymbol{E}}^{\text{in}}||^2$ does NOT fix a unique B-matrix. The solution of the B-matrix in this case is, in general, ambiguous.

- In order to select one possible solution for the B-matrix, additional conditions or constraints must be introduced.

- We assume that the B-matrix is diagonal and identical in the TE-TM coordinate system that is defined by the input and output wavevector directions.

- This is one typical way to define the B-matrix, but not the only way.

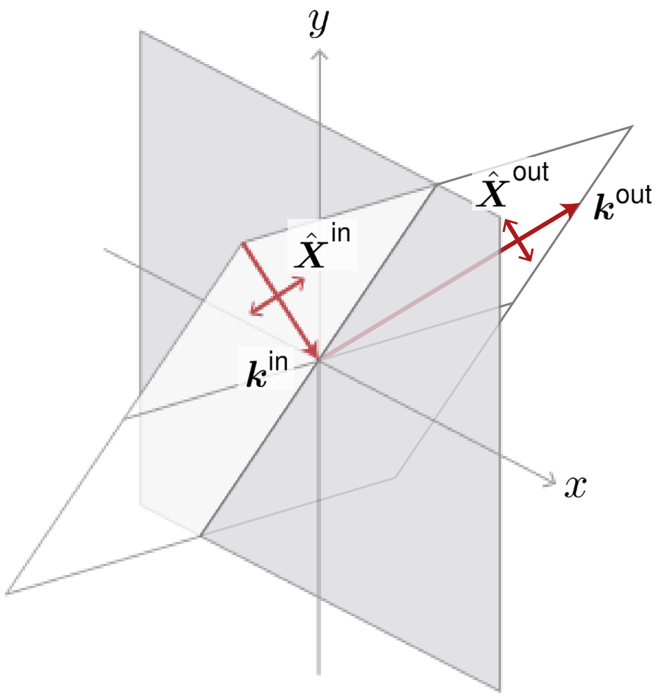
Function Algorithm – From Efficiency to B-Matrix

- We can generally define the basis vectors of the TE-TM coordinate system, by using the two wavevectors $\boldsymbol{k}^{\text{in}}$ and $\boldsymbol{k}^{\text{out}}$, as
  - TE (Y-direction) : $\hat{\boldsymbol{Y}} = \hat{\boldsymbol{k}}^{\text{in}} \times \hat{\boldsymbol{k}}^{\text{out}}$, and
  - TM (X-direction): $\hat{\boldsymbol{X}}^{\text{in/out}} = \text{sign}(\hat{\boldsymbol{k}}_z^{\text{in/out}}) \left(\hat{\boldsymbol{Y}} \times \hat{\boldsymbol{k}}^{\text{in/out}}\right)$.
- In the special case when the relation

$$\hat{\boldsymbol{k}}^{\text{in}} = \pm\hat{\boldsymbol{k}}^{\text{out}}$$

holds, we define the basis vectors as

  - TE (Y-direction) : $\hat{\boldsymbol{Y}} = \hat{\boldsymbol{k}}^{\text{in}} \times \hat{\boldsymbol{z}}$, with $\boldsymbol{z}$ as the unit direction vector along z-direction, and
  - TM (X-direction): $\hat{\boldsymbol{X}}^{\text{in/out}} = \text{sign}(\hat{\boldsymbol{k}}_z^{\text{in/out}}) \left(\hat{\boldsymbol{Y}} \times \hat{\boldsymbol{k}}^{\text{in/out}}\right)$.



definition of the basis vectors
for the TE-TM coordinate system

Function Algorithm – From Efficiency to B-Matrix

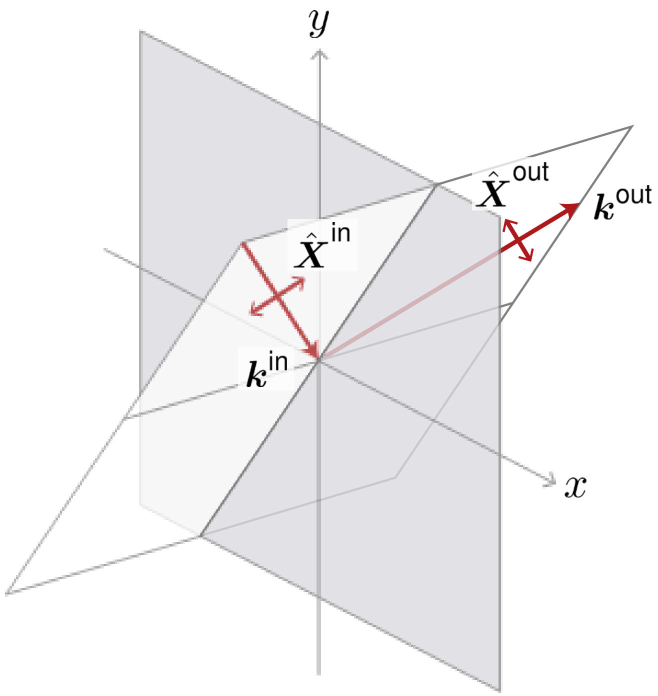
- With the basis vectors of the TE-TM coordinate system defined, we can write down the following transformation relations

$$\begin{pmatrix} \tilde{E}_x^{\text{in}} \\ \tilde{E}_y^{\text{in}} \end{pmatrix} = \begin{pmatrix} \hat{X}_x^{\text{in}} & \hat{Y}_x \\ \hat{X}_y^{\text{in}} & \hat{Y}_y \end{pmatrix} \begin{pmatrix} \tilde{E}_{\text{X}}^{\text{in}} \\ \tilde{E}_{\text{Y}}^{\text{in}} \end{pmatrix},$$

$$\begin{pmatrix} \tilde{E}_x^{\text{out}} \\ \tilde{E}_y^{\text{out}} \end{pmatrix} = \begin{pmatrix} \hat{X}_x^{\text{out}} & \hat{Y}_x \\ \hat{X}_y^{\text{out}} & \hat{Y}_y \end{pmatrix} \begin{pmatrix} \tilde{E}_{\text{X}}^{\text{out}} \\ \tilde{E}_{\text{Y}}^{\text{out}} \end{pmatrix}.$$

- In the TE-TM system, the B-matrix is expressed in the following simple form:

$$\begin{pmatrix} \tilde{E}_{\text{X}}^{\text{out}} \\ \tilde{E}_{\text{Y}}^{\text{out}} \end{pmatrix} = \begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \tilde{E}_{\text{X}}^{\text{in}} \\ \tilde{E}_{\text{Y}}^{\text{in}} \end{pmatrix}.$$



definition of the basis vectors
for the TE-TM coordinate system

Function Algorithm – From Efficiency to B-Matrix

- In the TE-TM coordinate, it is straight-forward to see that

$$||\tilde{\boldsymbol{E}}^{\text{in}}||^2 = |\tilde{E}_{\text{X}}^{\text{in}}|^2 + |\tilde{E}_{\text{Y}}^{\text{in}}|^2 \,, \quad ||\tilde{\boldsymbol{E}}^{\text{out}}||^2 = \left(|\tilde{E}_{\text{X}}^{\text{in}}|^2 + |\tilde{E}_{\text{Y}}^{\text{in}}|^2\right) |B|^2 \,,$$

  and by substituting in the expression of diffraction efficiency, we find that

$$\frac{||\tilde{\boldsymbol{E}}^{\text{out}}||^2}{||\tilde{\boldsymbol{E}}^{\text{in}}||^2} = |B|^2 = \frac{\mu^{\text{out}} k_z^{\text{in}}}{\mu^{\text{in}} k_z^{\text{out}}} \eta \,,$$

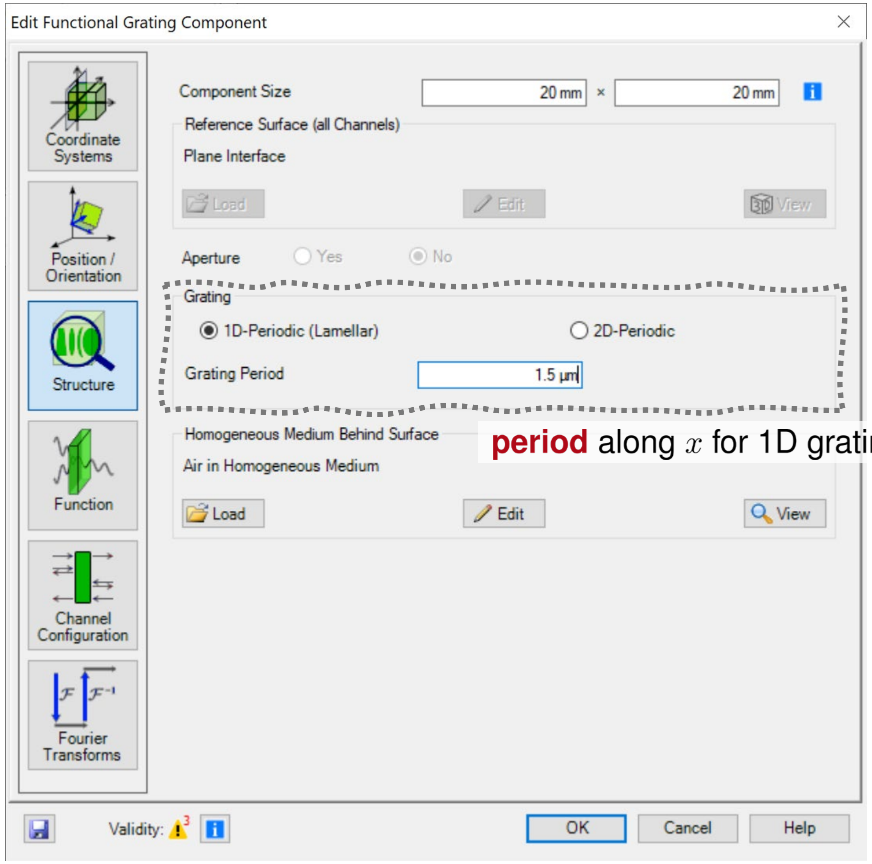
  from which the value of $|B|$ can be fixed.

- With the help of the coordinate transformation relations, we can finally write down the relations in the x-y coordinate system:

$$\begin{pmatrix} \tilde{E}_x^{\text{out}} \\ \tilde{E}_y^{\text{out}} \end{pmatrix} = B \begin{pmatrix} \hat{e}_x^{\text{X,out}} & \hat{e}_x^{\text{Y}} \\ \hat{e}_y^{\text{X,out}} & \hat{e}_y^{\text{Y}} \end{pmatrix} \begin{pmatrix} \hat{e}_x^{\text{X,in}} & \hat{e}_x^{\text{Y}} \\ \hat{e}_y^{\text{X,in}} & \hat{e}_y^{\text{Y}} \end{pmatrix}^{-1} \begin{pmatrix} \tilde{E}_x^{\text{in}} \\ \tilde{E}_y^{\text{in}} \end{pmatrix} .$$

Usage in VirtualLab Fusion

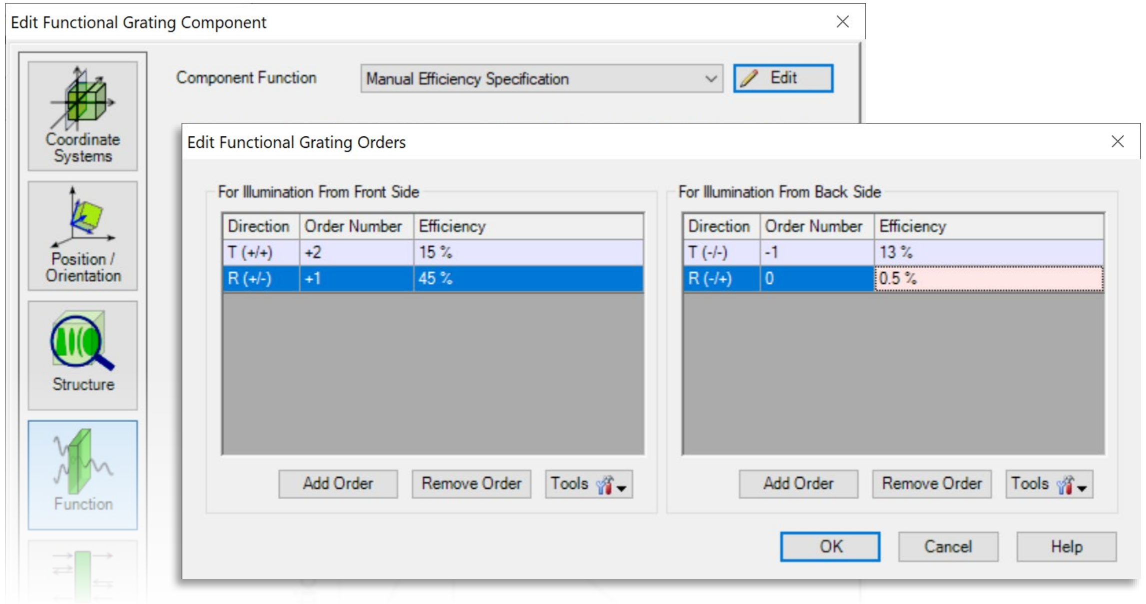
- Idealized grating functions do not require any knowledge about the specific shape of the grating structure, and can be initialized simply with the **period** value(s).



period along x for 1D gratings

Usage in VirtualLab Fusion

- Idealized grating functions do not require any knowledge about the specific shape of the grating structure, and can be initialized simply with the **period** value(s).
- Then, the **indices** of the diffraction orders to be considered and the corresponding **efficiencies** need to be specified in addition.
- The corresponding B-matrix is automatically derived from the diffraction efficiency, by assuming the B-matrix is diagonal and identical in the TE-TM coordiante system.



Document Information

title	VirtualLab Fusion Technology – Idealized Grating Functions
document code	TEC.0004
version	1.0
category	Technology White Paper
further reading	- VirtualLab Fusion Technology – FMM / RCWA [S-Matrix] - Grating Stretcher for Ultrashort Pulses