VirtualLab Fusion Technology – Solvers and Functions

Fresnel Matrix

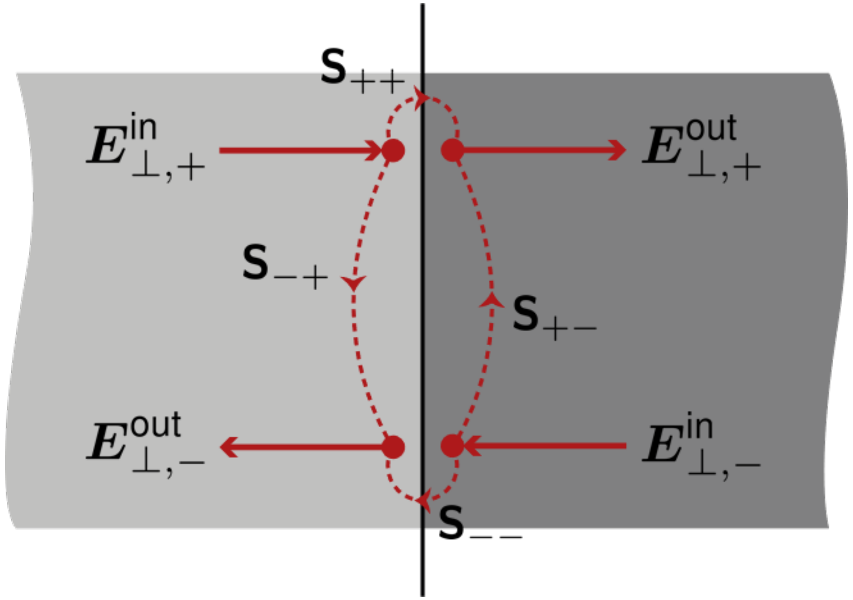
For the **Plane Surface Component**

Abstract

The Fresnel matrix solver works in the spatial frequency domain (**k domain**). It consists of

1. an eigenmode solver for the homogeneous media on both sides of the interface and
2. matching of the boundary conditions at the plane interface separating those two media.

The eigenmode solver computes the field solution in the k domain for the homogeneous medium in each layer, and then boundary conditions are applied to compute the matrix of reflection and transmission coefficients. In contrast to the traditional Fresnel coefficients (typically given for TE and TM, or s- and p-polarization), our solver gives the result corresponding to the E_x and E_y field components directly.



Solver Algorithm – Eigenmode Solver

- In the Fresnel matrix calculation, we deal with Maxwell's equations for homogeneous isotropic media, as written below:

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}k_0 \boldsymbol{H}(\boldsymbol{r})\,,$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}k_0 \epsilon \boldsymbol{E}(\boldsymbol{r})$$

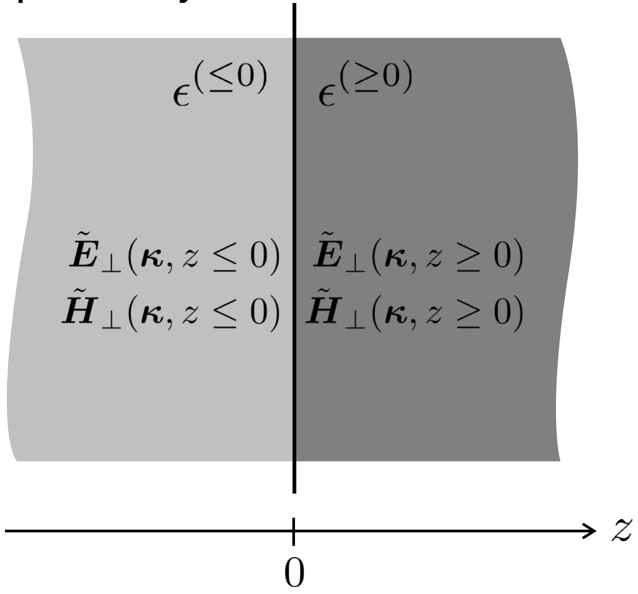
with constant **permittivity**, $\epsilon = \epsilon^{(\leq 0)}$ or $\epsilon^{(\geq 0)}$, on either side.

- The eigenmode solution in the k domain can be found via

$$\begin{pmatrix} \tilde{E}_x(\boldsymbol{\kappa}, z) \\ \tilde{E}_y(\boldsymbol{\kappa}, z) \\ \tilde{H}_x(\boldsymbol{\kappa}, z) \\ \tilde{H}_y(\boldsymbol{\kappa}, z) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\tilde{W}_B & -\tilde{W}_D & \tilde{W}_B & \tilde{W}_D \\ \tilde{W}_C & \tilde{W}_B & -\tilde{W}_C & -\tilde{W}_B \end{pmatrix} \begin{pmatrix} \tilde{C}_+^{\mathrm{I}} \exp(\gamma z) \\ \tilde{C}_+^{\mathrm{II}} \exp(\gamma z) \\ \tilde{C}_-^{\mathrm{I}} \exp(-\gamma z) \\ \tilde{C}_-^{\mathrm{II}} \exp(-\gamma z) \end{pmatrix},$$

with $\tilde{W}_B = \frac{n_x n_y}{n_z}$, $\tilde{W}_C = n_z + \frac{n_x^2}{n_z}$, $\tilde{W}_D = n_z + \frac{n_y^2}{n_z}$, $n_x = \frac{k_x}{k_0}$, $n_y = \frac{k_y}{k_0}$, and $n_z = (\epsilon\mu - n_x^2 - n_y^2)^{1/2}$, $\gamma = \mathrm{i}k_0 n_z$.

- Here we use $\boldsymbol{r} = (x, y, z)$ and $\boldsymbol{\rho} = (x, y)$ as the 3D position vector and its 2D projection onto the transversal plane respectively.



Solver Algorithm – Boundary Conditions

- At the surface (i.e. $z = 0$ position), based on the boundary conditions, we can directly write down the following relation:

$$\begin{pmatrix} \mathbf{W}_{11}^{(\leq 0)} & \mathbf{W}_{12}^{(\leq 0)} \\ \mathbf{W}_{21}^{(\leq 0)} & \mathbf{W}_{22}^{(\leq 0)} \end{pmatrix} \begin{pmatrix} C_{+}^{(\leq 0)} \\ C_{-}^{(\leq 0)} \end{pmatrix} = \begin{pmatrix} \mathbf{W}_{11}^{(\geq 0)} & \mathbf{W}_{12}^{(\geq 0)} \\ \mathbf{W}_{21}^{(\geq 0)} & \mathbf{W}_{22}^{(\geq 0)} \end{pmatrix} \begin{pmatrix} C_{+}^{(\geq 0)} \\ C_{-}^{(\geq 0)} \end{pmatrix} .$$

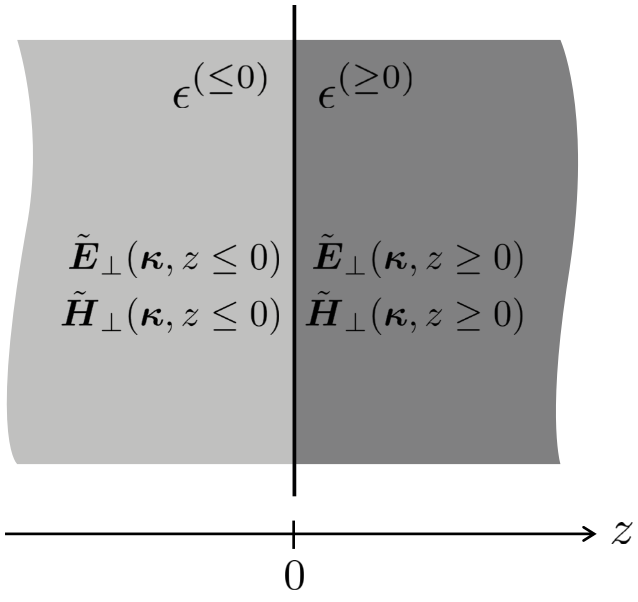
- After some algebraic manipulations, we finally obtain

$$\begin{pmatrix} C_{+}^{(\geq 0)} \\ C_{-}^{(\leq 0)} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix} \begin{pmatrix} C_{+}^{(\leq 0)} \\ C_{-}^{(\geq 0)} \end{pmatrix} .$$

- Considering input from $z \leq 0$ side only, we have

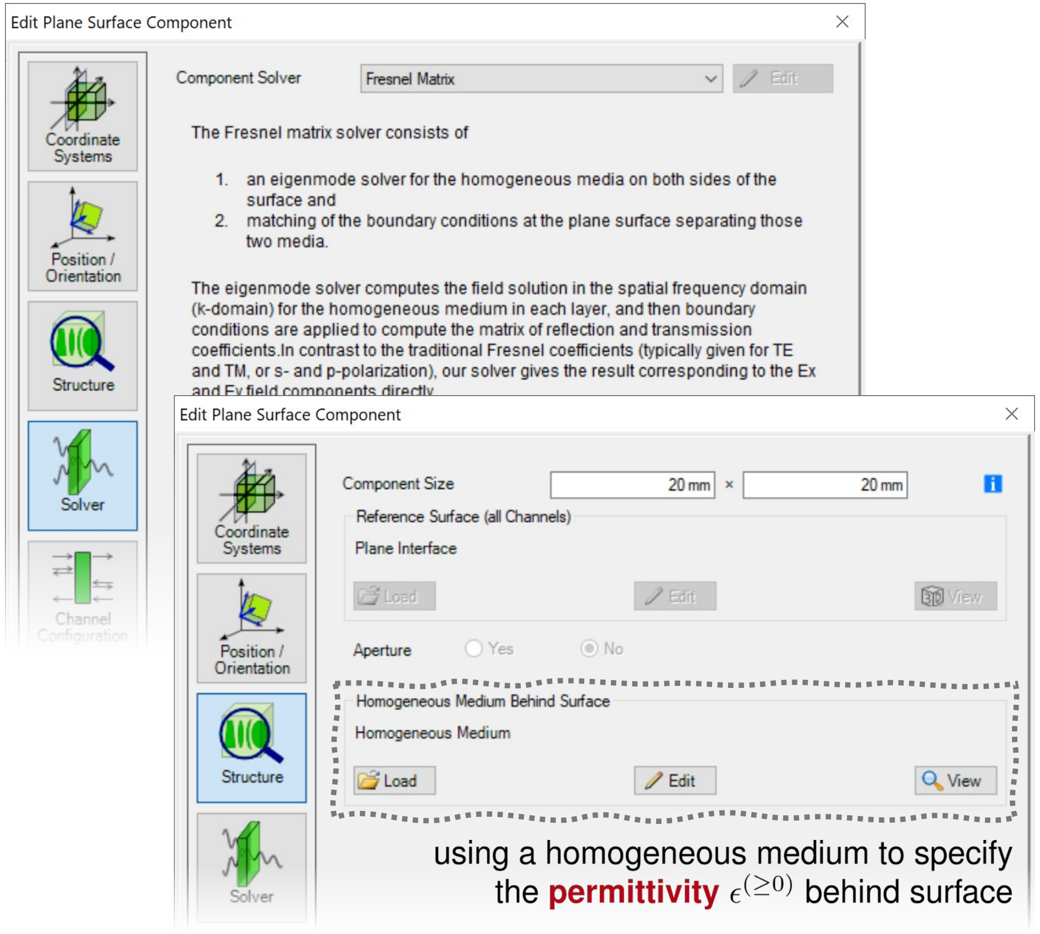
$$\mathbf{S}_{11}(\boldsymbol{\kappa}) = \mathbf{T} = \begin{pmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{pmatrix} , \quad \text{and} \quad \mathbf{S}_{21}(\boldsymbol{\kappa}) = \mathbf{R} = \begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} ,$$

as the matrices of transmission and reflection coefficients.



Usage in VirtualLab Fusion

- Take the Plane Surface Component as an example:
  - the **permittivity** $\epsilon^{(\leq 0)}$ in front of the surface is determined by the preceding optical setup;
  - the **permittivity** $\epsilon^{(\geq 0)}$ behind the surface is specified by a homogeneous isotropic medium.
- The Fresnel matrix is calculated for each spatial frequency $\boldsymbol{\kappa}$ contained in the arbitrary input field which reaches the plane surface.



using a homogeneous medium to specify the **permittivity** $\epsilon^{(\geq 0)}$ behind surface

Document Information

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