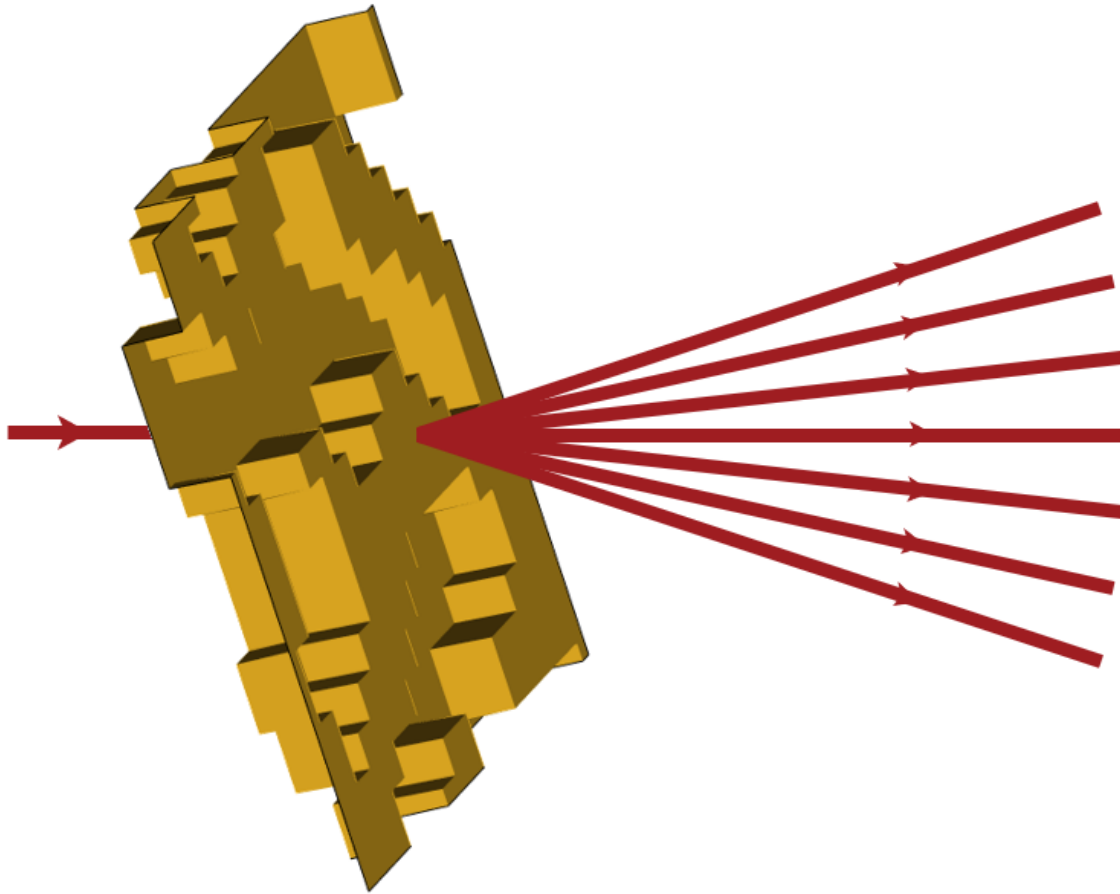


# **Design and Rigorous Analysis of Non-Paraxial Diffractive Beam Splitter**

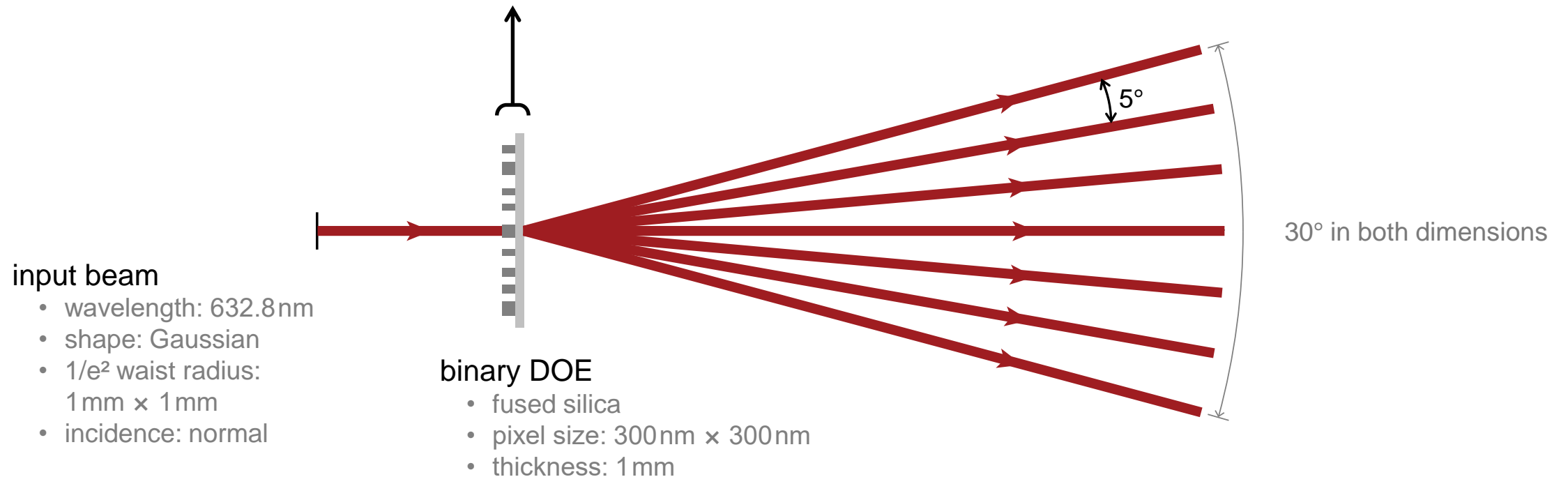
# Abstract



The direct design of non-paraxial diffractive beam splitters is still a challenge. Due to the quite large diffraction angle, the feature size of the element become similar to the wavelength of light. Hence, the typically used paraxial modeling approaches become inaccurate and rigorous techniques are required. Thus, in this example, the iterative Fourier transform algorithm (IFTA) and the thin element approximation (TEA) are used for the initial design of the diffractive optical element (DOE), and the Fourier modal method (FMM) also known as rigorous coupled wave analysis (RCWA) is applied afterwards for a rigorous performance evaluation, including the investigation of merit function changes in the case of height variations.

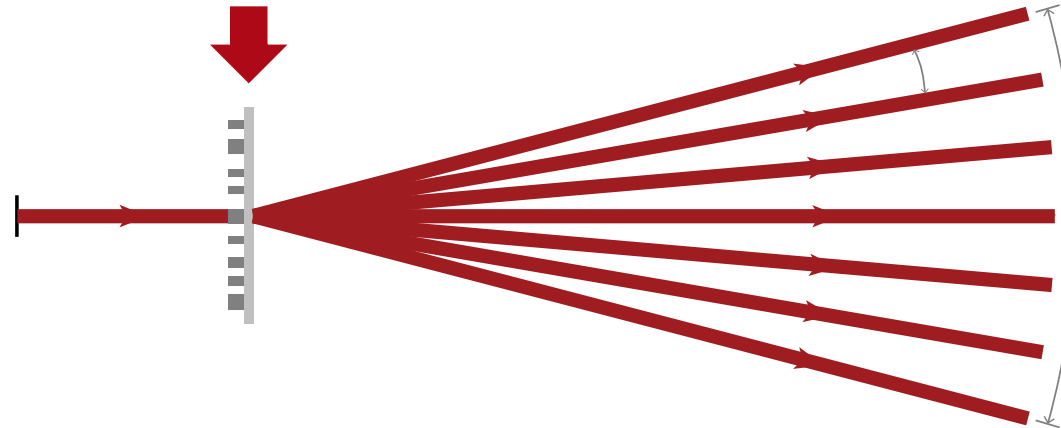
# Task

- initial design of a diffractive 1:7×7 beam splitter using a paraxial approximation (TEA) for the structure design part
- performance analysis and further optimization of uniformity and influence of zeroth order by using rigorous analyses (FMM/RCWA)





# **Simulation & Setup: Introduction of Tools & General Process Overview**

# Connected Modeling Techniques: Diffractive Beam Splitter

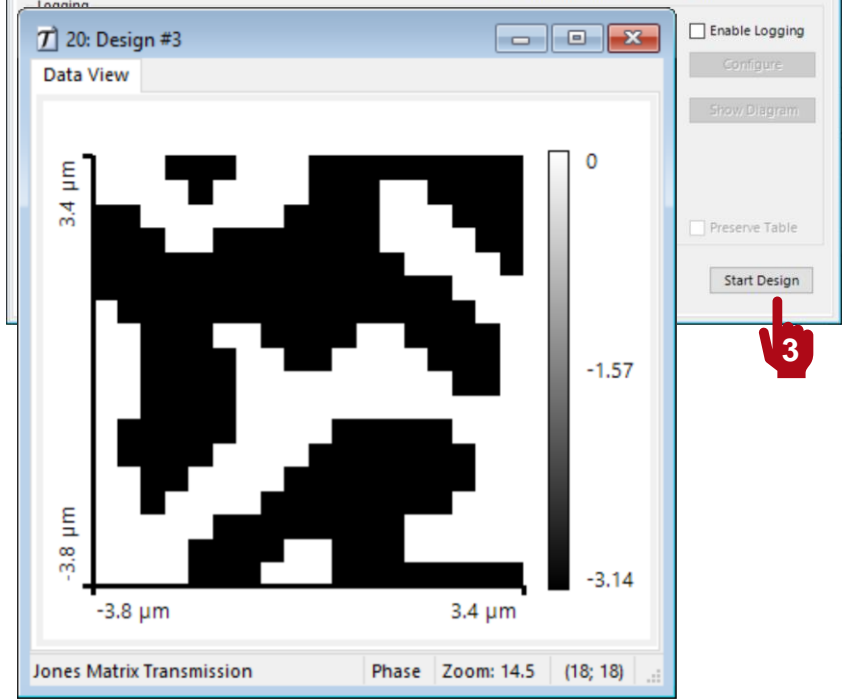
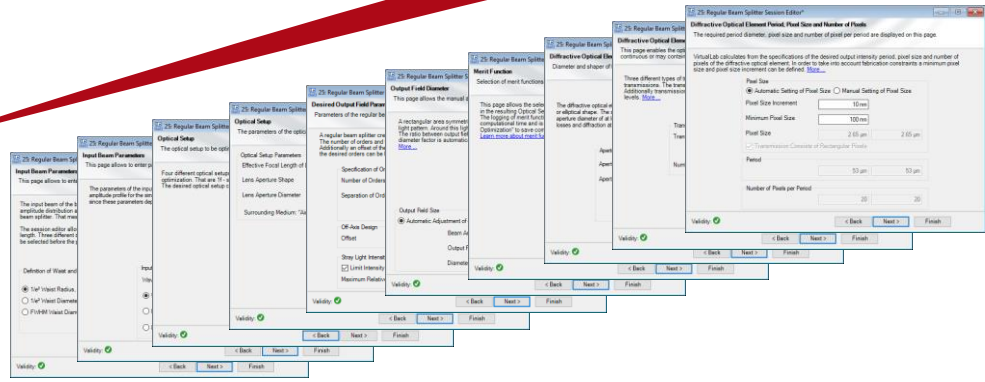
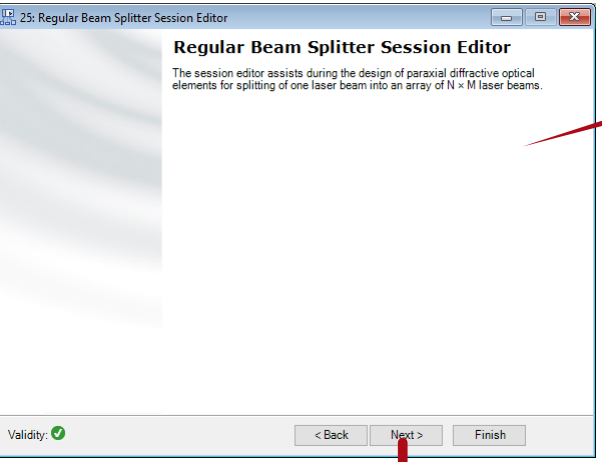
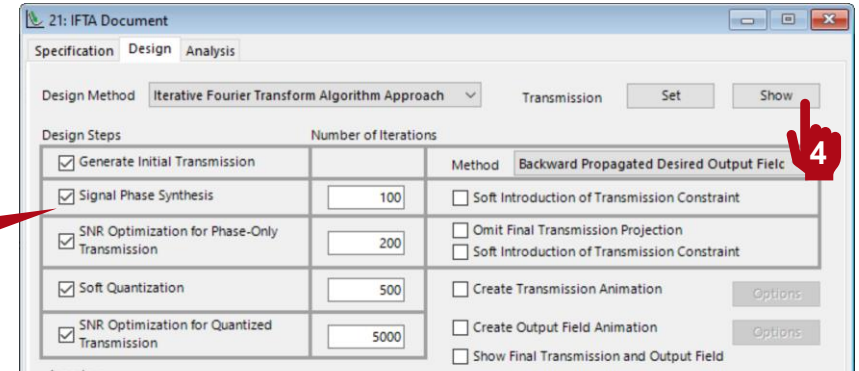
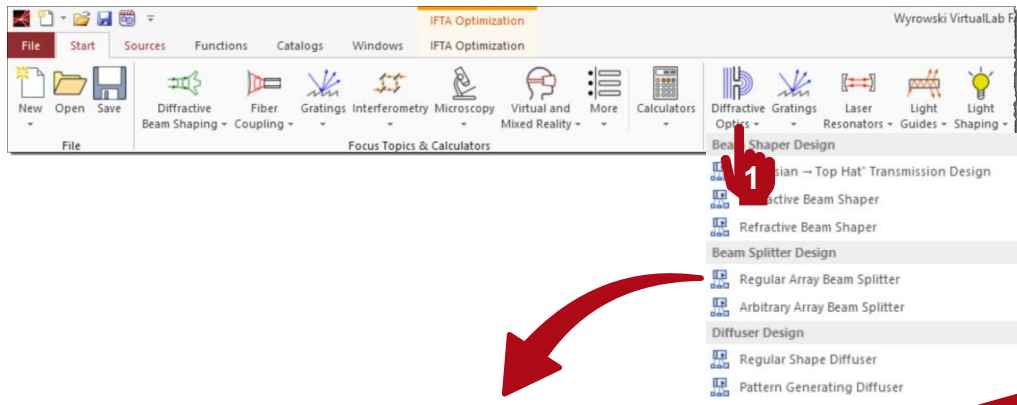


Available modeling techniques for microstructures:

Methods	Preconditions	Accuracy	Speed	Comments
Functional Approach	-	low	very high	diffraction angles acc. to grating equation; manual efficiencies
Thin Element Approximation (TEA)	smallest features $> \sim 10\lambda$	high	very high	inaccurate for larger NA and thick elements; x-domain
	smallest features $< \sim 2\lambda$	low	very high	
Fourier Modal Method (FMM)	period $< \sim (5\lambda \times 5\lambda)$	very high	high	rigorous solution; fast for structures and periods similar to the wavelength; more demanding for larger periods; k-domain
	period $> \sim (15\lambda \times 15\lambda)$	very high	slow	

 In this example we want to investigate the difference between the **Thin Element Approximation (TEA)** and the **Fourier Modal Method (FMM)** for a real beam splitter.
   


# Via Configuration Assistant & IFTA to a Phase Design



With the *Regular Beam Splitter Session Editor*, VirtualLab Fusion offers a step-by-step assistant for the configuration of the design/optimization document (IFTA tool) for the design of a diffractive splitter.

# Convert Transmission Function To Structure

- The resulting transmission function can be converted into a structure profile by applying *Structure Design* from the *Design* ribbon.
- For this conversion, the thin element approximation (TEA) is used. The resulting height profile is therefore proportional to the initial phase function.
- VirtualLab Fusion delivers the calculated structure data in the form of already preset elements of an optical setup.
- To use the designed structure in different simulation scenarios either the sampled surface or the specified stack needs to be taken from within the component.

Structure Design

Diffractive Optical Element (DOE) (for Field Tracing)

Plane Surface 1 Z: 0 mm

Tracing 2 Z: 1 mm

Edit Diffractive Optical Element (DOE) Component

Operator Specified by ...  Complex Surface Response  Stack

Grating Stack

Sampled Grating

On Front Side of Base Surface  On Back Side of Base Surface

Method for Stack Analysis  Parabasal Thin Element Approximation

Index	z-Distance	z-Position	Surface	Subsequent Medium	Comments
1	0 mm	0 mm	Sampled Surface	Air in Homogeneous...	Enter your comment

Validity:

Periodicity & Aperture

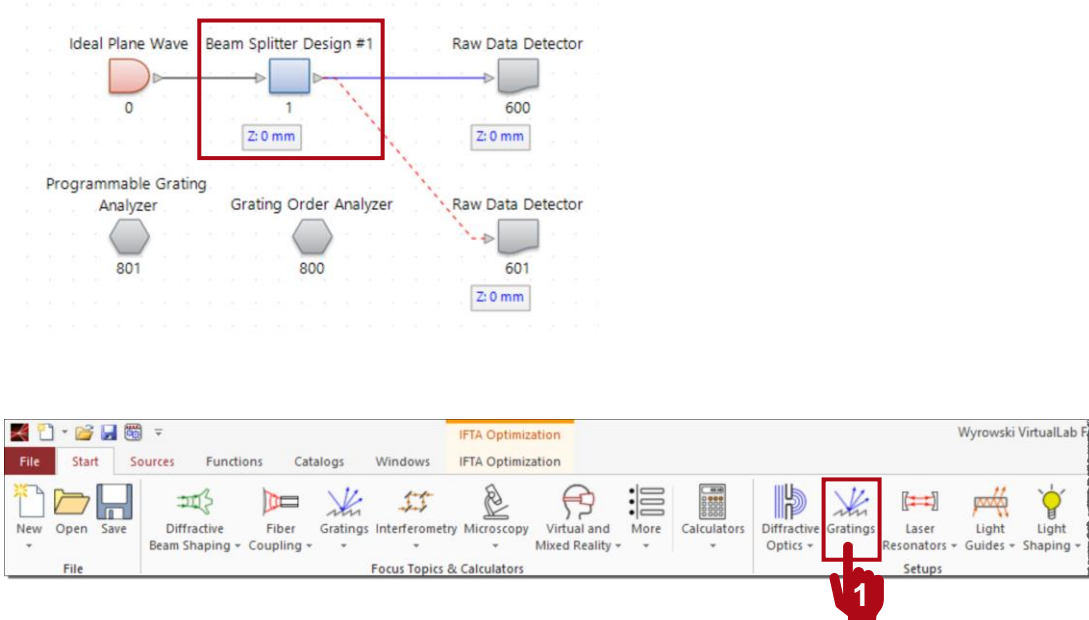
Periodic  Non-Periodic

Stack Period is  with Index

Stack Period  ×

Here we save the stack for further use.

# Diffractive Beam Splitter Surface



For further evaluation, a *General Grating Optical Setup* is used, where the previously saved stack is loaded. The *Grating Optical Setup* offers unique tools, components and analyzers to further investigate the characteristics and performance of a given periodic structure.

The 'Edit General Grating Component' dialog box is shown. It has two tabs: '1D-Periodic (Lamellar)' and '2D-Periodic', with '2D-Periodic' selected. The 'Base Block' is 'Fused Silica in Homogeneous Medium' with a thickness of '10 mm'. The 'Stacks' section has 'Use Stack on First Surface' checked. The 'Catalog Entry' is 'DOE 2'. A red arrow points to the 'Edit' button in the 'Catalog Entry' section. The 'Edit Stack' sub-dialog is open, showing a table of stack layers and grating parameters.

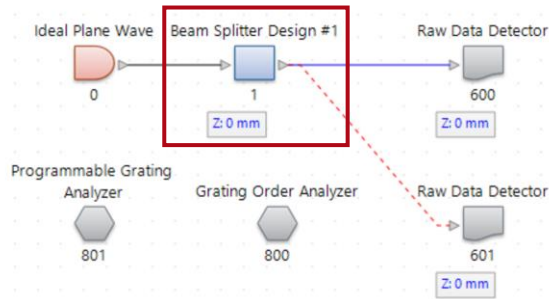
Index	z-Distance	z-Position	Surface	Subsequent Medium	Com
1	0 mm	0 mm	Sampled Surface	Air in Homogeneous	Enter your commen

Validity:  Preview Wavelength: 632.8 nm

Periodicity & Aperture:  Periodic  Non-Periodic  
Stack Period is: Dependent from the Period of Surface with Index: 1  
Stack Period: 7.2  $\mu\text{m}$  × 7.2  $\mu\text{m}$



# Diffraction Beam Solvers – TEA & FMM

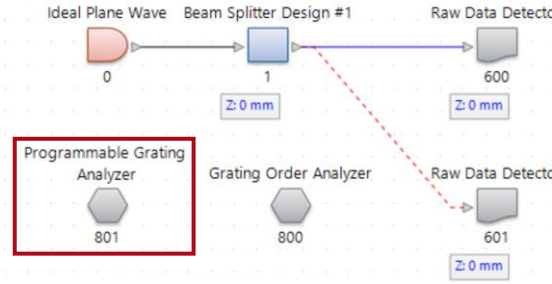
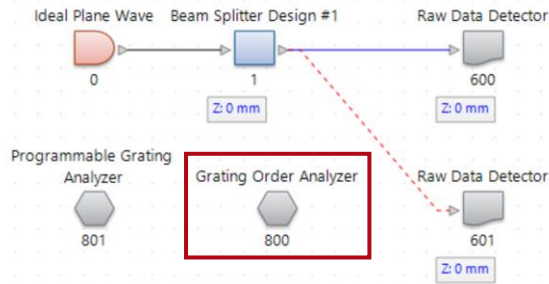
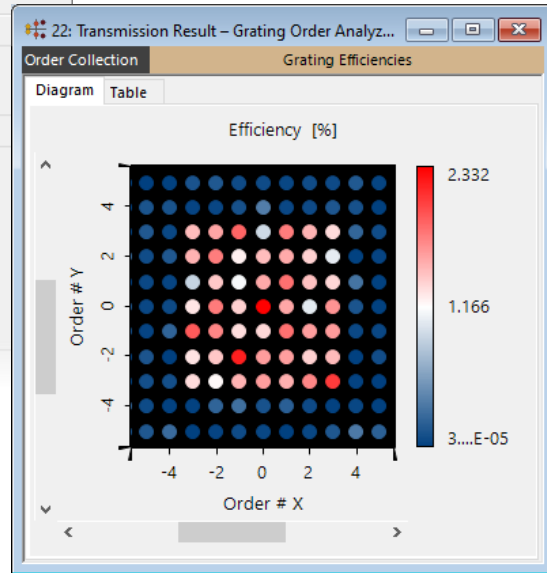


- The *General Grating Component* offers the *Thin Element Approximation (TEA)* and the *Fourier Modal Method (FMM)* as solvers to model the given grating.
- The *TEA* usually generates results faster but may have accuracy issues, if the structures are smaller than about 5 to 10 times the wavelength.
- The *FMM* allows for a rigorous simulation but requires a higher numerical effort.

Component Propagation: Fourier Modal Method (selected), Fourier Modal Method, Thin Element Approximation (pointed to by a red hand icon). Edit button.

Interface	Stack	Medium
1	Plane Interface	Fused Silica in Homogen...
	Fourier Modal Met	Fourier Modal Met
2	Plane Interface	Fused_Silica in Homogen...
	Fourier Modal Met	

# Grating Order & Programmable Grating Analyzer

```

43 #region Main method
44 DetectorResultObject[] detectorResults = new DetectorResultObject[7];
45
46 // initialization
47 double totalEfficiency = 0.0;
48 double minEfficiency = double.PositiveInfinity;
49 double maxEfficiency = double.NegativeInfinity;
50 double minEfficiency_withoutZero = double.PositiveInfinity;
51 double maxEfficiency_withoutZero = double.NegativeInfinity;
52 double zerothEfficiency = 0.0;
53 // loop
54 for (int iY = 0; iY < NumberOfOrdersY; iY++) {
55     for (int iX = 0; iX < NumberOfOrdersX; iX++) {
56         int OrderIndexX = -(NumberOfOrdersX - 1) / 2 + iX; // counting
57         int OrderIndexY = -(NumberOfOrdersY - 1) / 2 + iY; // counting
58         OrderInfo currentOrderInformation = TransmissionResults.GetOrd
    
```

The Programmable Grating Analyzer is a tool, that allows for more specific outputs, e.g.:

- total efficiency
- uniformity error
- evaluations of certain orders
- ...

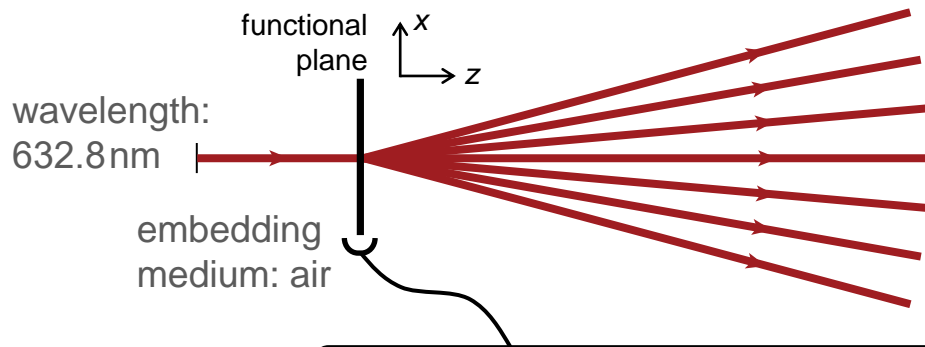
Detector	Sub - Detector	Result
Grating Analyzer	Value #6: Uniformity Error (RMS)	67.453014 %
Grating Analyzer	Value #5: Zeroth Order Error	451.46414 %
Grating Analyzer	Value #4: Zeroth Efficiency	7.9549562 %
Grating Analyzer	Value #3: Average Efficiency without Zeroth Order	1.3068398 %
Grating Analyzer	Value #2: Average Efficiency	1.4425156 %
Grating Analyzer	Value #1: Total Efficiency	70.683265 %

## **Designs & Evaluation Results**

- Phase Function Designs
- Structure Designs
- TEA Evaluation
- FMM Evaluation
- Height Scaling Check (for Optimization/Tolerancing)

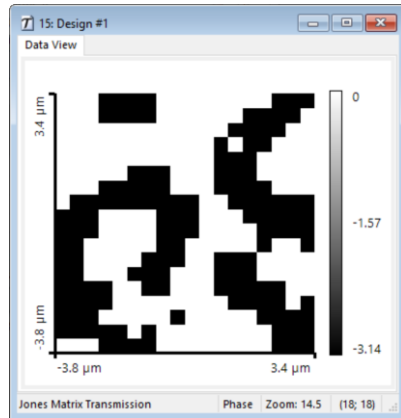
# Phase-Only Transmission Design

In this step, the iterative Fourier transform algorithm (IFTA) is applied for a binary phase-only transmission design.

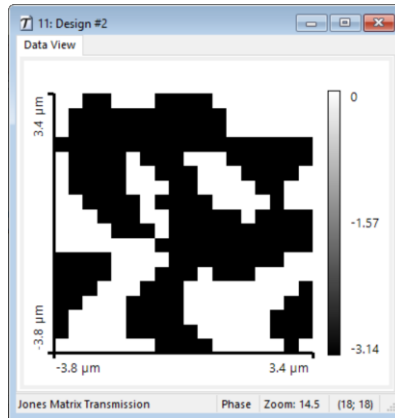


VirtualLab Fusion offers the *Multiple Run* document, which allows the user to perform an arbitrary number of designs with an option to filter the results according to certain criteria. The following three results were obtained this way; we will evaluate them further.

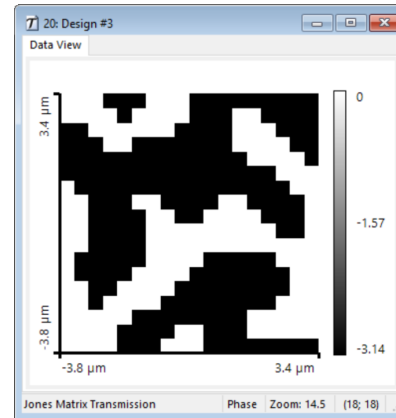
phase functions



design #1



design #2



design #3

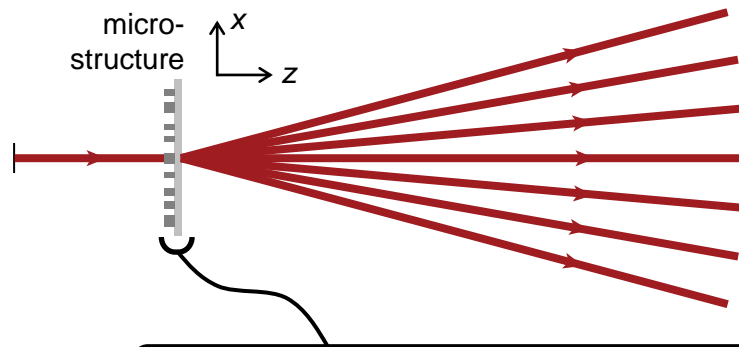
...

Element's Features

- period size: 7.2 nm ( $\sim 11.4\lambda$ )
- smallest structure size (sampling distance): 400 nm ( $\sim 0.6\lambda$ )

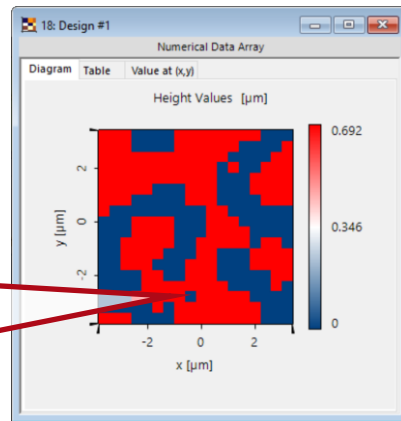
# Structure Design

Next, the thin element approximation (TEA) is used for the structure design of the height profile, i.e., under a paraxial assumption (the phase function and the resulting height profile are therefore proportional).

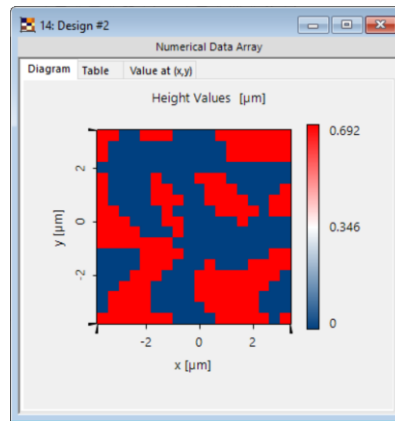


automatic conversion from phase-only transmission to structure height profile, according to given wavelength and material

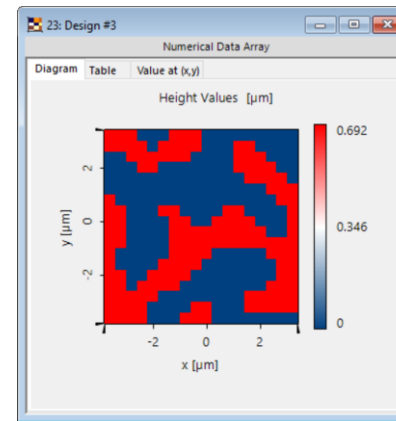
height profiles



design #1



design #2

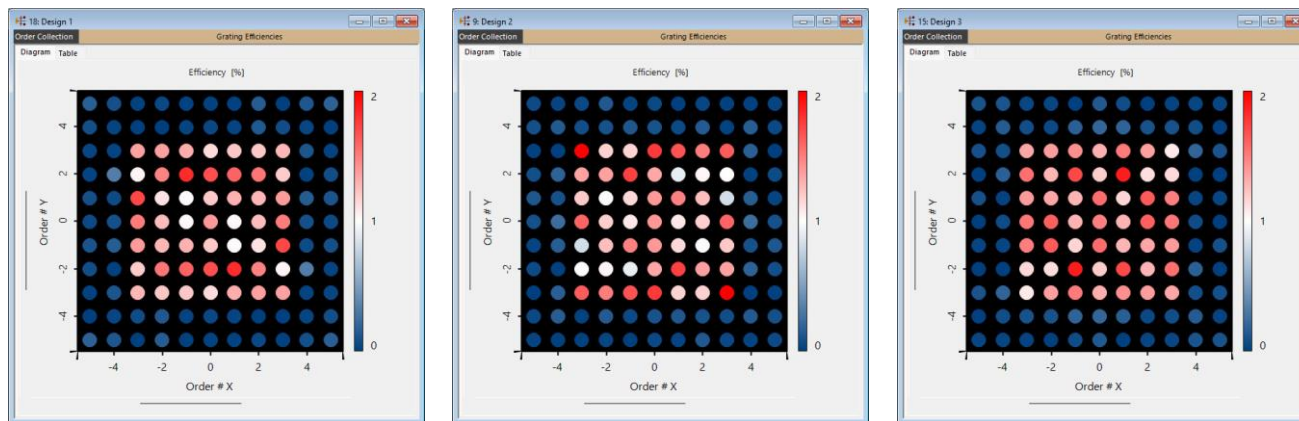
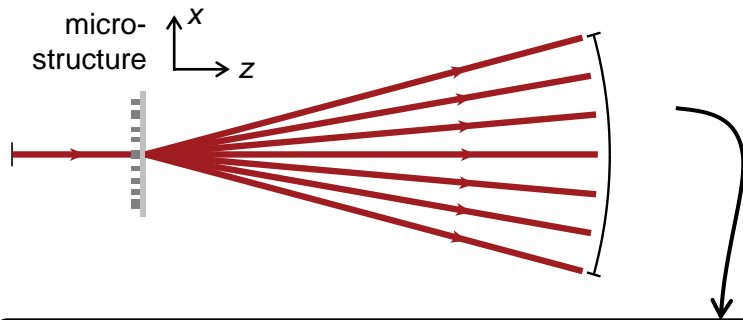


design #3

**Note:** Such small features might cause issues for fabrication and achieving good results.

# Performance Evaluation with TEA

Now, the obtained microstructure is evaluated by applying TEA, which was also used for the structure design and is accurate under paraxial conditions.



Merit Function	Design #1	Design #2	Design #3
Total Efficiency	66.1%	65.7%	69.5%
Average Efficiency (of use orders)	1.4%	1.3%	1.4%
Zereth Order Efficiency	1.4%	1.4%	1.5%
Zereth Order Error*	<b>4.3%</b>	5.7%	6.4%
Uniformity Error**	28.7%	41.3%	<b>27.1%</b>
Uniformity Error without 0 <sup>th</sup> Order	28.7%	41.3%	27.1%

$$* \text{ Zereth Order Error} = \frac{\text{Zereth Efficiency} - \text{Average Efficiency}}{\text{Average Efficiency}}$$

$$** \text{ Uniformity Error} = \frac{\text{Max. Efficiency} - \text{Min. Efficiency}}{\text{Max. Efficiency} + \text{Min. Efficiency}}$$

From the results obtained by TEA, system #1 and #3 look roughly similar. Design #2 shows significantly larger uniformity errors. Furthermore:

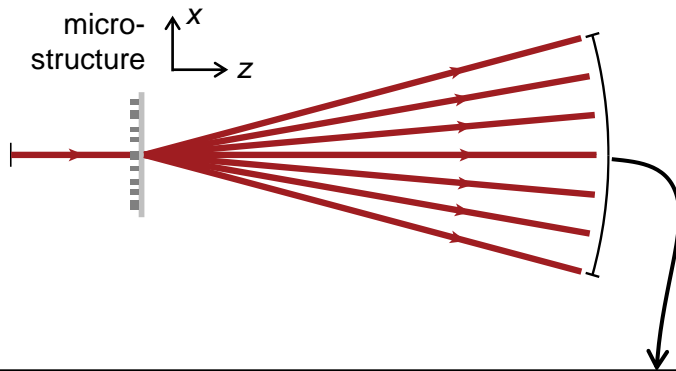
- For all three design the 0<sup>th</sup> order never stands out.
- Design #1 has the lowest zereth order error.
- Design #3 has the lowest uniformity error.

But these values are not expected to be accurate, since the assumptions of the paraxial model do not hold.

→ A rigorous analysis is urgently required.

# Performance Evaluation with FMM

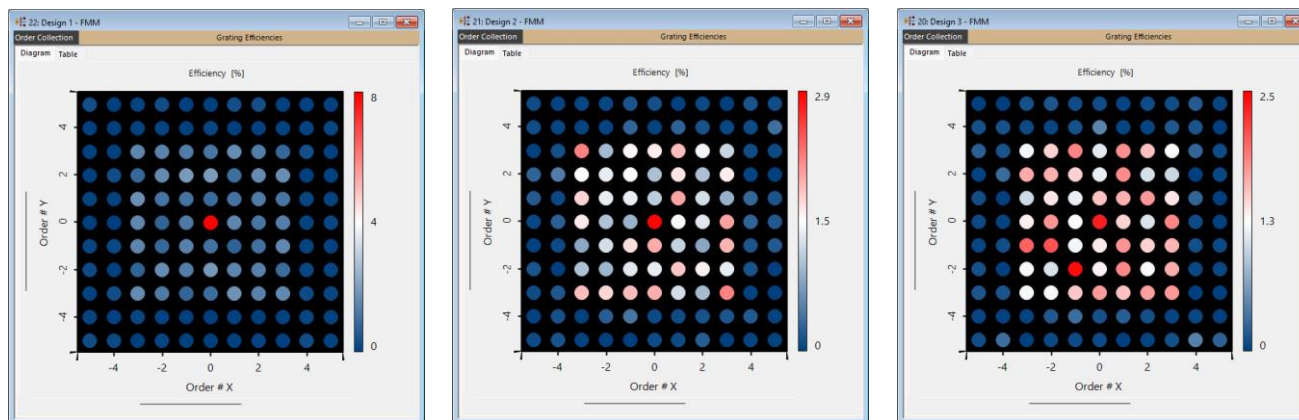
After the investigation with TEA, a rigorous analysis by using FMM is performed.



Merit Function	Design #1	Design #2	Design #3
Total Efficiency	70.5%	70.2%	73.8%
Average Efficiency (of use orders)	1.4%	1.4%	1.5%
Zerth Order Efficiency	8.0%	2.9%	2.3%
Zerth Order Error*	<b>453.3%</b>	100.4%	55.6%
Uniformity Error**	82.8%	56.8%	<b>42.4%</b>
Uniformity Error without 0 <sup>th</sup> Order	44.1%	46.3%	42.4%

$$* \text{ Zerth Order Error} = \frac{\text{Zerth Efficiency} - \text{Average Efficiency}}{\text{Average Efficiency}}$$

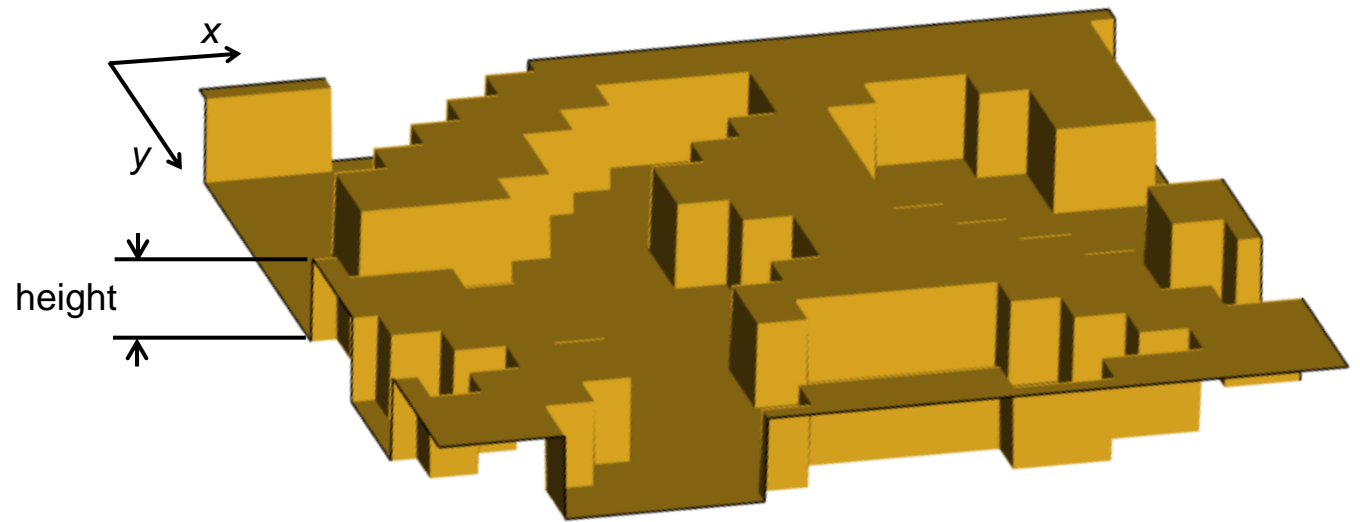
$$** \text{ Uniformity Error} = \frac{\text{Max. Efficiency} - \text{Min. Efficiency}}{\text{Max. Efficiency} + \text{Min. Efficiency}}$$



- Rigorously evaluated (FMM), it turns out that design #1 actually produces the strongest 0<sup>th</sup> diffraction order, resulting in a very poor uniformity.
- The designs seem to have a comparable *Uniformity Error without 0<sup>th</sup> Order*.
- Therefore, an optimization to minimize the *0<sup>th</sup> Order Error* may improve the performance distinctly.

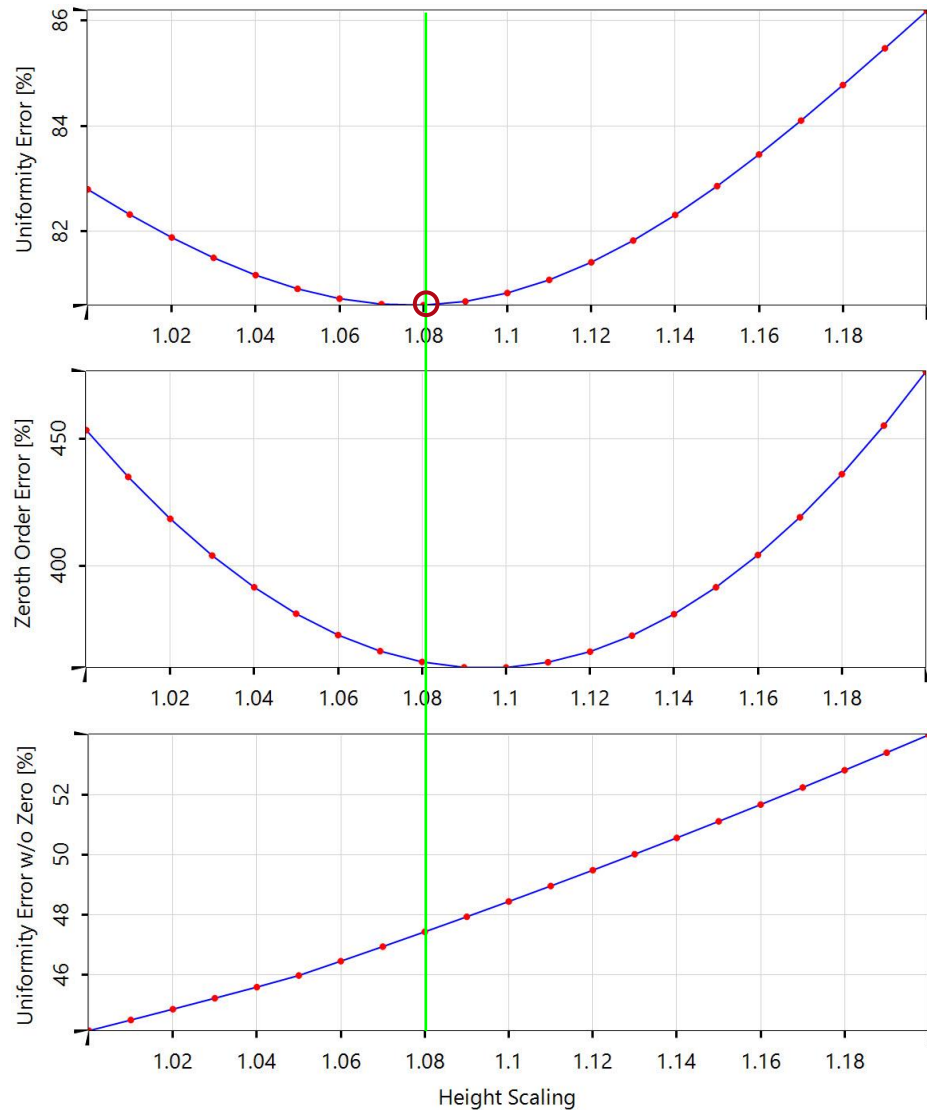
# Further Analyses (Post-Optimization, Tolerancing)

- A scaling of the height profile has a strong influence on the 0<sup>th</sup> order's efficiency.
- This can be used to correct an undesired efficiency of the zeroth order and thus also to improve the uniformity.
- The *Parameter Run* document is the best suited tool to perform such investigations.
- At the same time, such simulations with varied heights may serve as tolerance investigation.





# Further Optimization – Zeroth Order Tuning Design #1

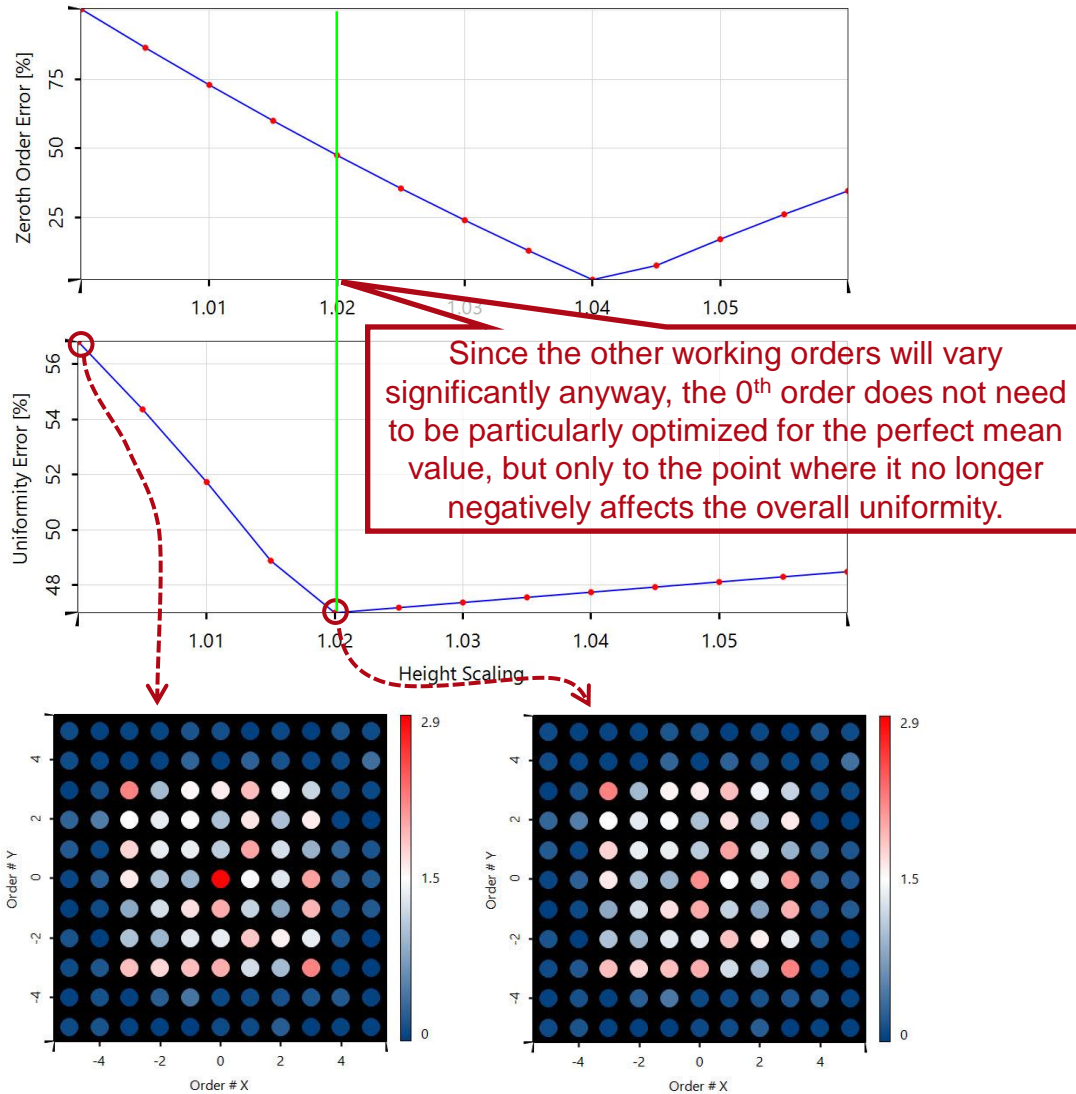


- It turns out that simple height scaling does not sufficiently reduce the high value of the *Zeroth Order Error* of design #1.
- It should be noted, that while the goal of height scaling is the reduction of the 0<sup>th</sup> order, the uniformity error and other merit functions are also affected, albeit to a lesser extent.

Comparison between initial vs post-optimized design aiming at low *Uniformity Error*

Merit Function	Design #1	with scaling factor 1.08
Total Efficiency	70.5%	67.87%
Average Efficiency (of use orders)	1.4%	1.4%
Zeroth Order Efficiency	8.0%	6.4%
Zeroth Order Error	<b>453.3%</b>	<b>362.6%</b>
Uniformity Error	82.8%	80.6%
Uniformity Error without 0 <sup>th</sup> Order	44.1%	47.4%

# Further Optimization – Zeroth Order Tuning Design #2

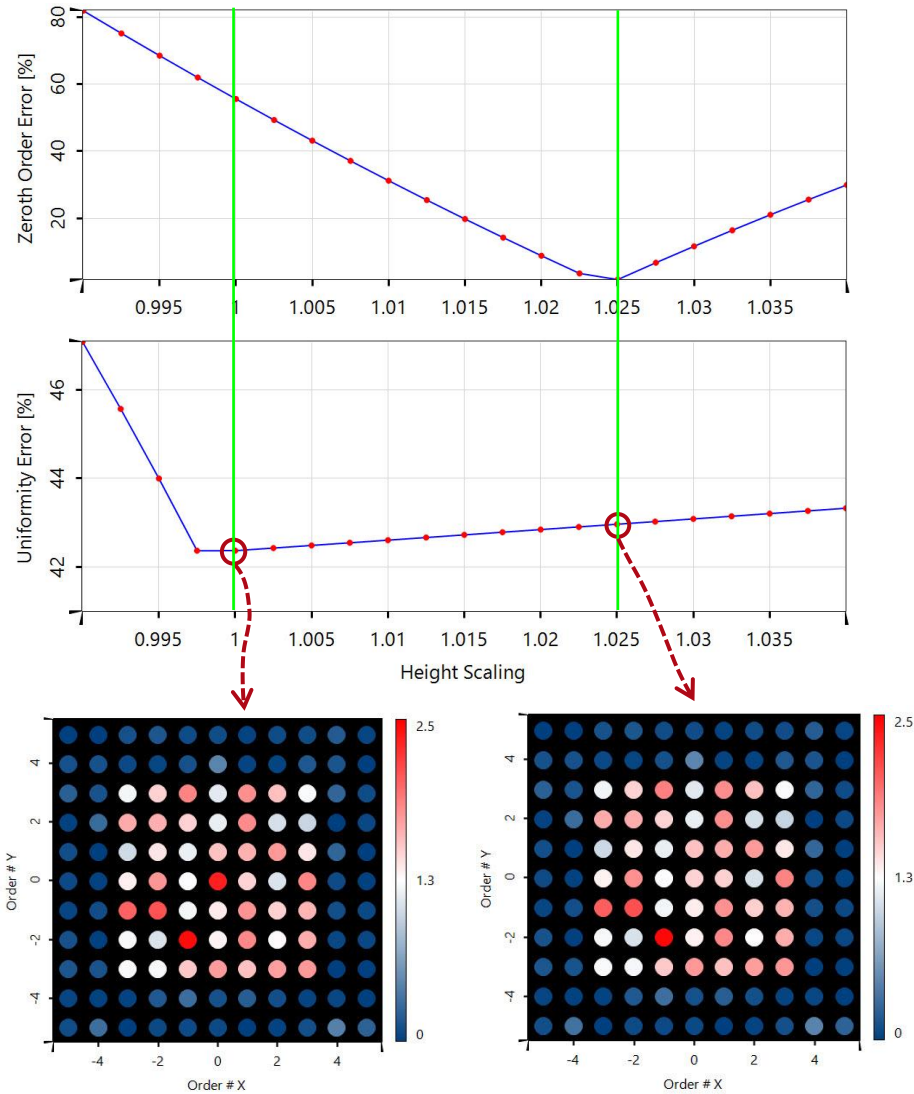


- The 0<sup>th</sup> order of design #2 is also distinctly higher but not as dominant as for design #1. Here a scaling might show more promising results.
- On the other hand, the height scaling won't optimize the merit function *Uniformity Error without 0<sup>th</sup> Order*. Hence in general, the best that can be expected, is a similar overall uniformity of all working orders including the 0<sup>th</sup> order.
- Usually, the other merit function values get worse, but not always. In any case, it is up to the optical engineer to decide which compromise is best.

Comparison between initial vs post-optimized design aiming at low *Uniformity Error*

Merit Function	Design #2	with scaling factor 1.02
Total Efficiency	70.2%	69.4%
Average Efficiency (of use orders)	1.4%	1.4%
Zeroth Order Efficiency	2.9%	2.1%
Zeroth Order Error	100.4%	45.2%
Uniformity Error	<b>56.8%</b>	<b>47.0%</b>
Uniformity Error without 0 <sup>th</sup> Order	46.3%	47.0%

# Further Optimization – Zeroth Order Tuning Design #3



- For this design, the height scaling shows no further improvement of the uniformity error.
- Nevertheless, a variation of the height scaling is still advisable, as it provides information on how sensitive the design is with respect to possible tolerances of the etching depth. In particular, we see that a larger structure height will slightly decrease the overall uniformity but allows much more stable results.

Comparison between initial vs post-optimized design aiming at low *Uniformity Error*

Merit Function	Design #3	with scaling factor 1.025
Total Efficiency	73.8%	72.9%
Average Efficiency (of use orders)	1.5%	1.5%
Zeroth Order Efficiency	2.3%	1.4%
Zeroth Order Error	<b>55.6%</b>	<b>1.7%</b>
Uniformity Error	<b>42.4%</b>	<b>42.9%</b>
Uniformity Error without 0 <sup>th</sup> Order	42.4%	42.9%

# Document Information

title	Design and Rigorous Analysis of Non-Paraxial Diffractive Beam Splitter
document code	DOE.0004
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software version	2023.1 (Build 1.556)
category	Application Use Case
further reading	<ul style="list-style-type: none"><li>- <a href="#"><u>Grating Order Analyzer</u></a></li><li>- <a href="#"><u>Configuration of Grating Structures by Using Interfaces</u></a></li><li>- <a href="#"><u>Design of a High-NA Beam Splitter with 24000 Dots Random Pattern</u></a></li><li>- <a href="#"><u>Design of Diffractive Beam Splitters for Generating a 2D Light Mark</u></a></li><li>- <a href="#"><u>High NA Splitter Optimization with User-Defined Merit Functions</u></a></li></ul>