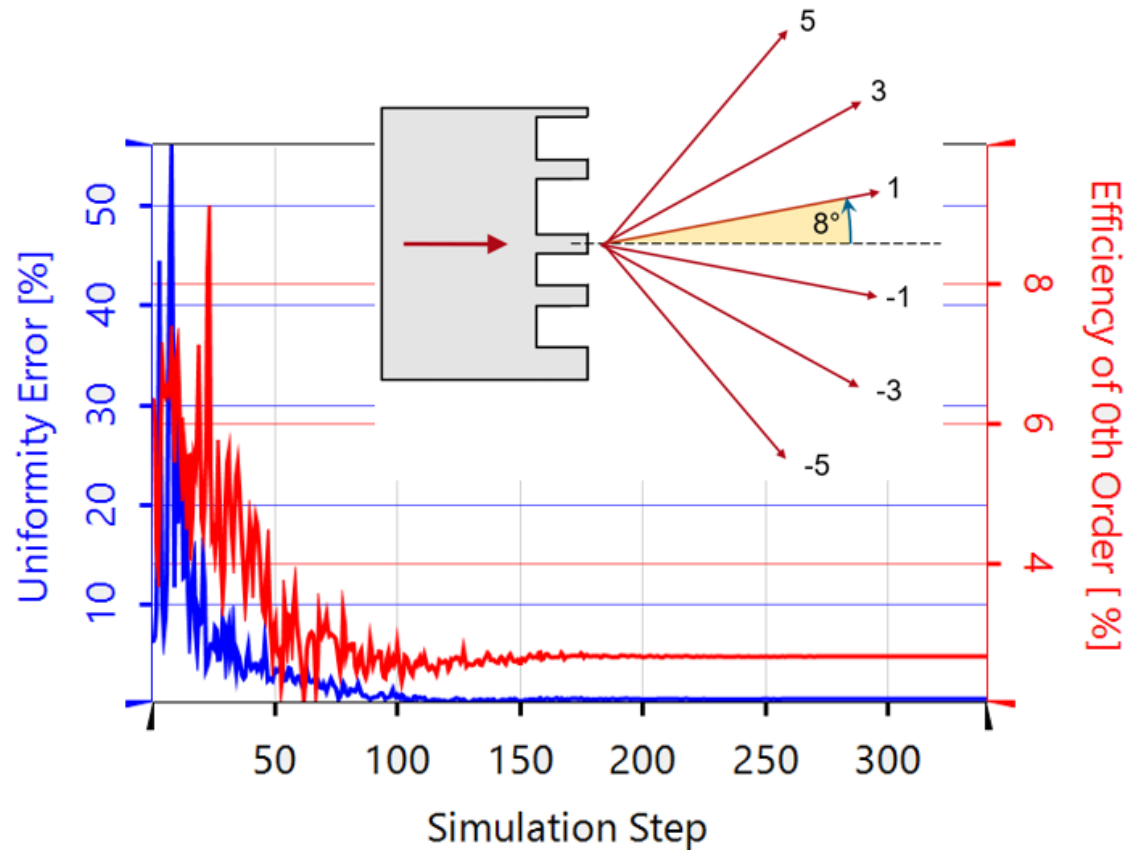


# High-NA Beam Splitter Optimization with User-Defined Merit Functions

# Abstract



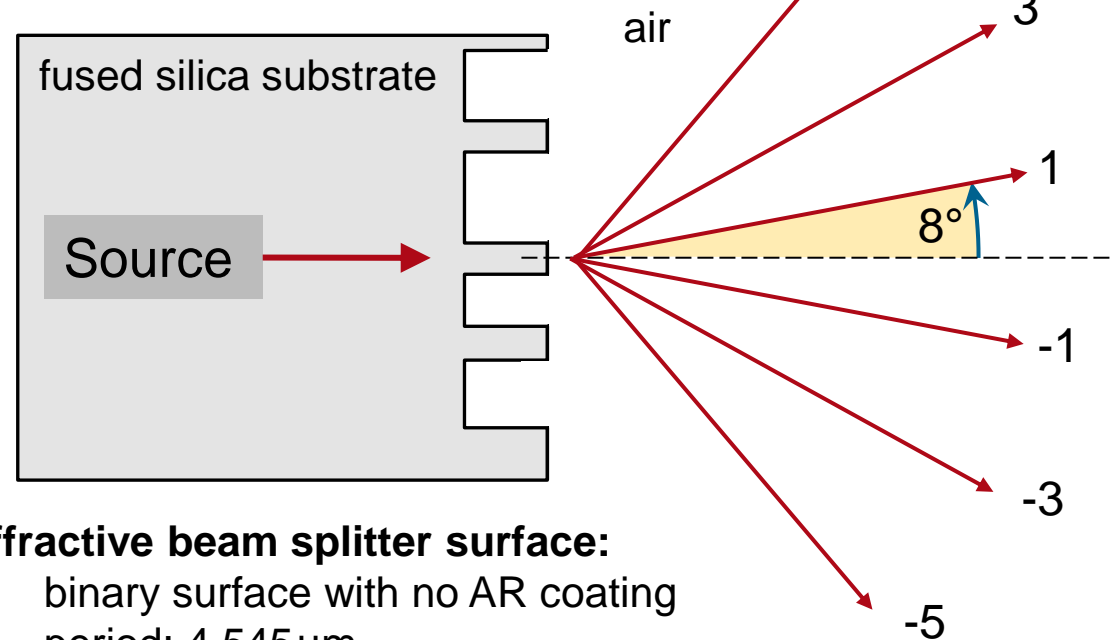
Diffractive beam splitters are often designed by applying certain paraxial approximations due to the direct relation between phase and structure and vice versa, which these algorithms provide. In case of non-paraxial or even high-NA splitters these approximations will introduce some inaccuracy and hence at least a rigorous analysis is advised, if not an additional rigorous post-optimization. In this use case, such rigorous evaluations are performed for an exemplary binary 1:6 splitter, using the odd diffraction orders. For this purpose, the structure of the initial system is parametrized, and a set of user-defined merit functions are defined via the Programmable Grating Analyzer. For the parametric optimization and subsequent tolerance analysis, the rigorous Fourier Modal Method (FMM) is used.

# Modeling Task

How to optimize the surface profile of the following diffractive high-NA 1:6 beam splitter to achieve optimal uniformity of the desired working orders?

## light parameters

- wavelength: 632.63nm
- polarization: along x-direction

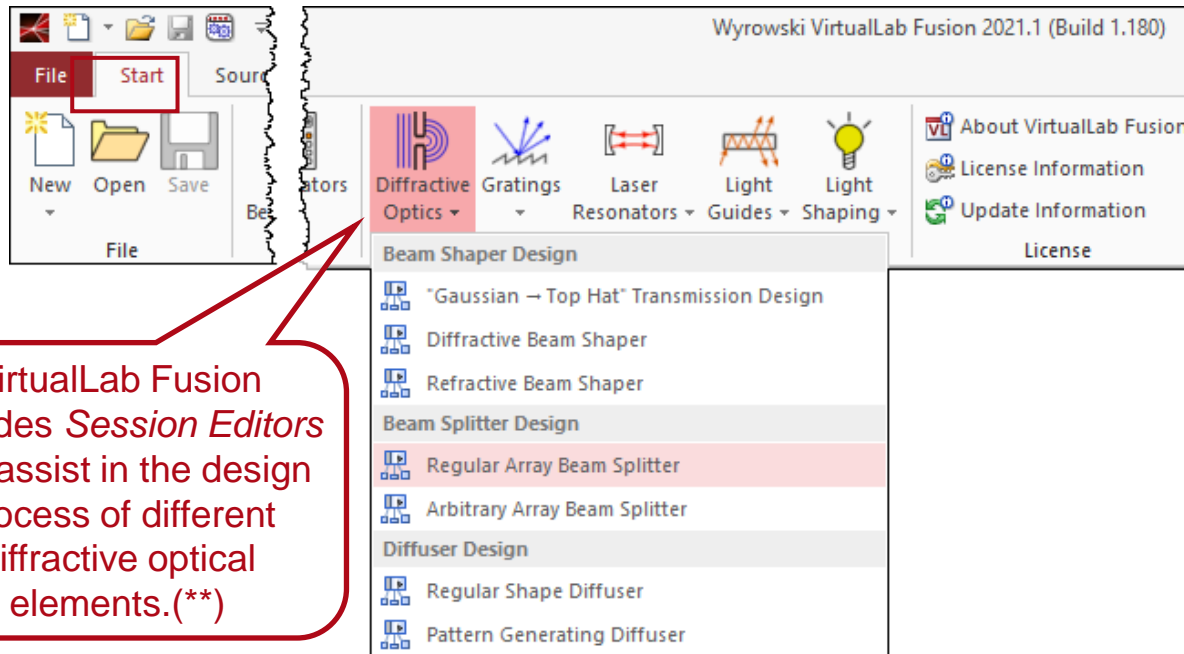


6 diffraction orders  
with uniform diffraction  
efficiencies

## diffractive beam splitter surface:

- binary surface with no AR coating
- period: 4.545 $\mu\text{m}$
- minimum feature size: 431 nm

# Initial Design of Diffractive Beam Splitter Surface(\*)

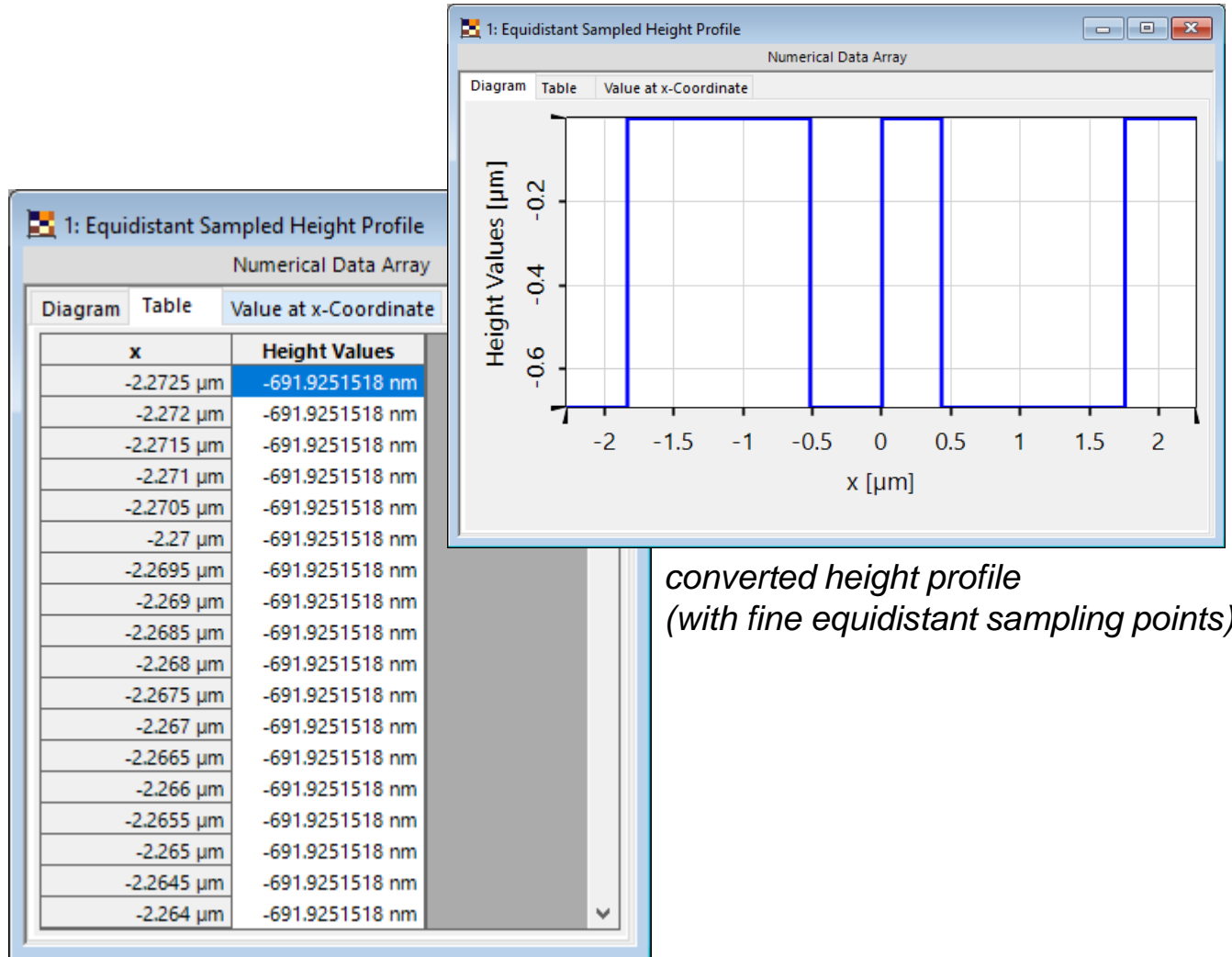


1. The initial beam splitter phase function was calculated by VirtualLab Fusion's Iterative Fourier Transform Algorithm (IFTA) design tool.
2. For the conversion to a height profile, a structure design based on the Thin Element Approximation (TEA) was applied.

(\*) not part of this use case

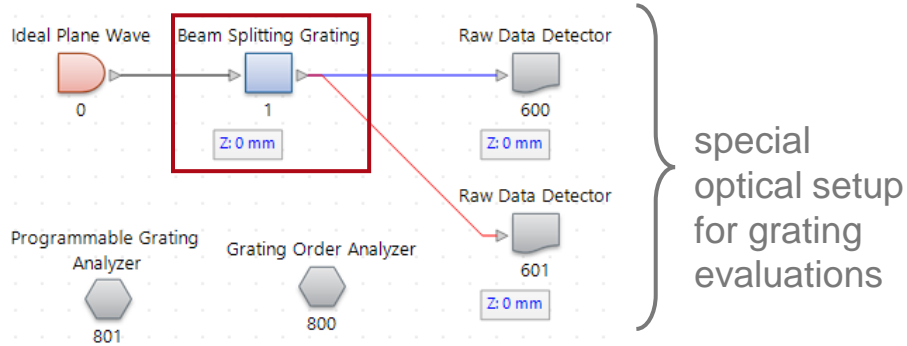
(\*\*) These Session Editors are available with the Diffractive Optics Toolbox Silver.

# Limitations of TEA and an Equidistant Sampled Structure



- TEA is well suited if the smallest feature sizes are not smaller than  $\sim 5$  times the wavelength. If this is not the case, the amplitude/phase distribution after interacting with the designed height profile might exhibit relevant deviations from the desired values.
- Thus, a rigorous evaluation is needed.
- And for a parametric optimization, the structure data needs to be defined differently.

# Data Preparation (Parametrization) for Post-Optimization



- For a rigorous analysis with the Fourier Modal Method (FMM), the sampled height profile from the structure design can directly be used.
- However, for a parametric optimization, the structure needs to be parametrized so that a suitable set of parameters can be used for the optimization.
- For this purpose, a VirtualLab Fusion module is used which converts the equidistantly sampled surface data into a non-equidistant transition point list (included in attached sample files).

1: Equidistant Sampled Height Profile

Numerical Data Array

x	Height Values
-2.2725 μm	-691.9251518 nm
-2.272 μm	-691.9251518 nm
-2.2715 μm	-691.9251518 nm
-2.271 μm	-691.9251518 nm
-2.2705 μm	-691.9251518 nm
-2.27 μm	-691.9251518 nm
-2.2695 μm	-691.9251518 nm
-2.269 μm	-691.9251518 nm
-2.2685 μm	-691.9251518 nm
-2.268 μm	-691.9251518 nm
-2.2675 μm	-691.9251518 nm
-2.267 μm	-691.9251518 nm
-2.2665 μm	-691.9251518 nm
-2.266 μm	-691.9251518 nm
-2.2655 μm	-691.9251518 nm
-2.265 μm	-691.9251518 nm
-2.2645 μm	-691.9251518 nm
-2.264 μm	-691.9251518 nm

C# module to convert sampled surface to transition point list

2: Module\_GeneratingTransitionPointData.cs

```

Source Code
Advanced Settings

public void Run() {
    int dummy = -1;
    dataArray1D daEquidistant = Globals.ActiveDocumentHistory.BrowseLastDocuments(DocumentFilter.F
        out dummy,
        "Select 1D Data

    if (!daEquidistant.IsEquidistant) {
        throw new Exception("Data Array has to be equidistant");
    }

    if (daEquidistant.InterpolationMethodForEquidistantSampling != InterpolationMethod.Nearest) {
        throw new Exception("Interpolation method of Data Array has to be nearest neighbor.");
    }

    List<VectorD> transitionPoints = new List<VectorD>();

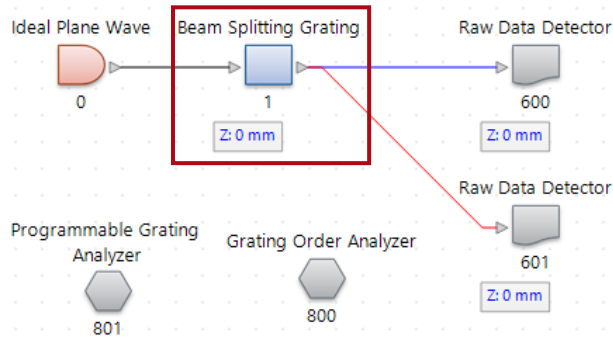
    double firstTransition = daEquidistant.CoordinateOfFirstDataPoint - (daEquidistant.SamplingDis

    //run through data points
    for (int runDataPoints = 0; runDataPoints < daEquidistant.NoOfDataPoints; runDataPoints++) {
        double currValue = daEquidistant.Data[0][runDataPoints].Re;
        bool addTransition
    }
}
    
```

Edit Transition Point List Surface

x	Height Values
-2.27275 μm	-691.9251518 nm
-1.782174663 μm	0 m
-490.8492518 nm	-691.9251518 nm
-290.278077 μm	0 m
440.3012811 nm	-691.9251518 nm
1.832171025 μm	0 m

# Diffractive Beam Splitter Surface for Further Optimization

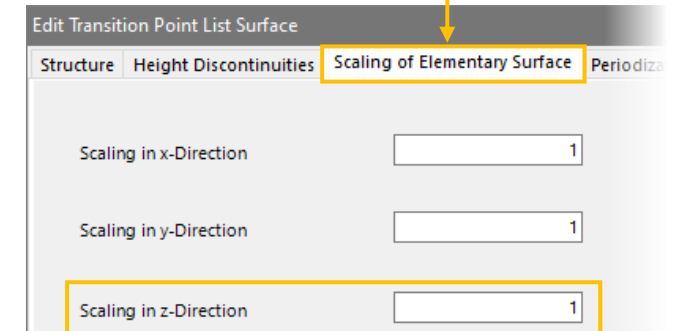
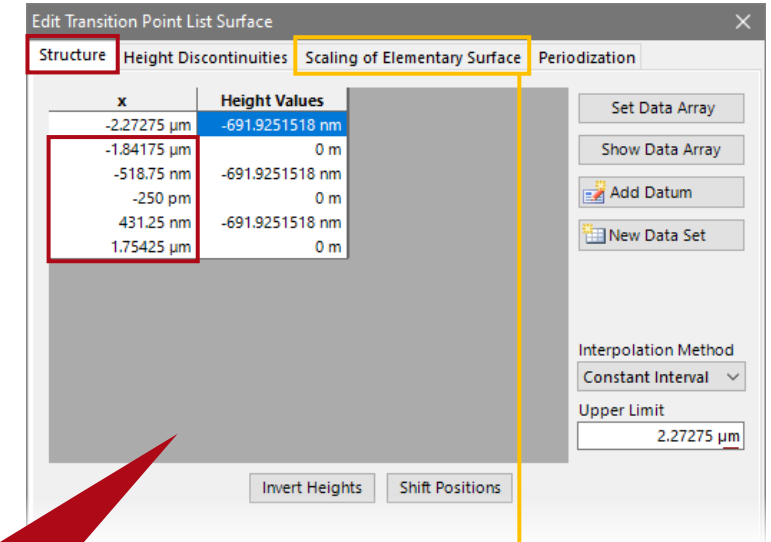
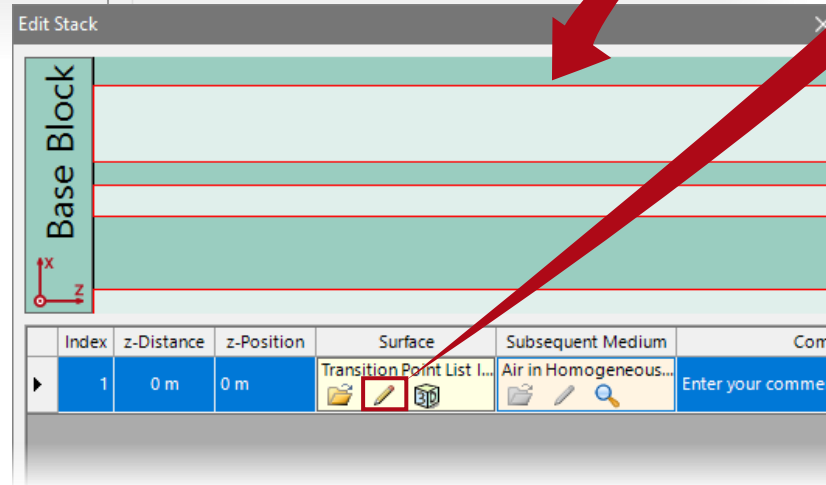
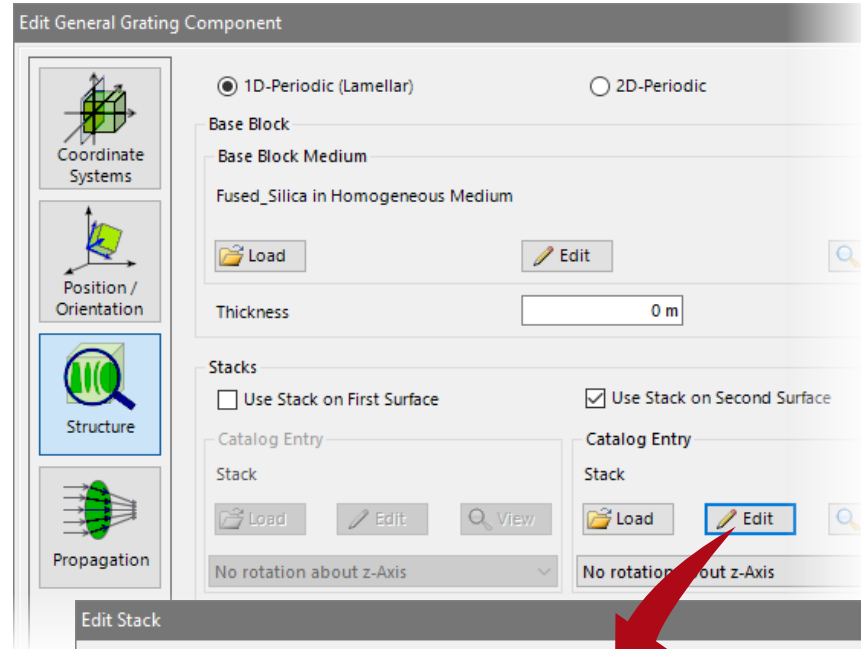


For the parametric optimization we plan to use

- the **position of the transition points(\*)**
- and the **z-scaling factor** (i.e. the profile height)

as free parameters.

(\*) except for the first one, which defines the border or the element



# Which Merit Functions for Which Diffraction Orders?

For the optimization it is not just relevant to have a well-parametrized structure, it is also important to define suitable merit functions, which are calculated based on distinct diffraction order results.

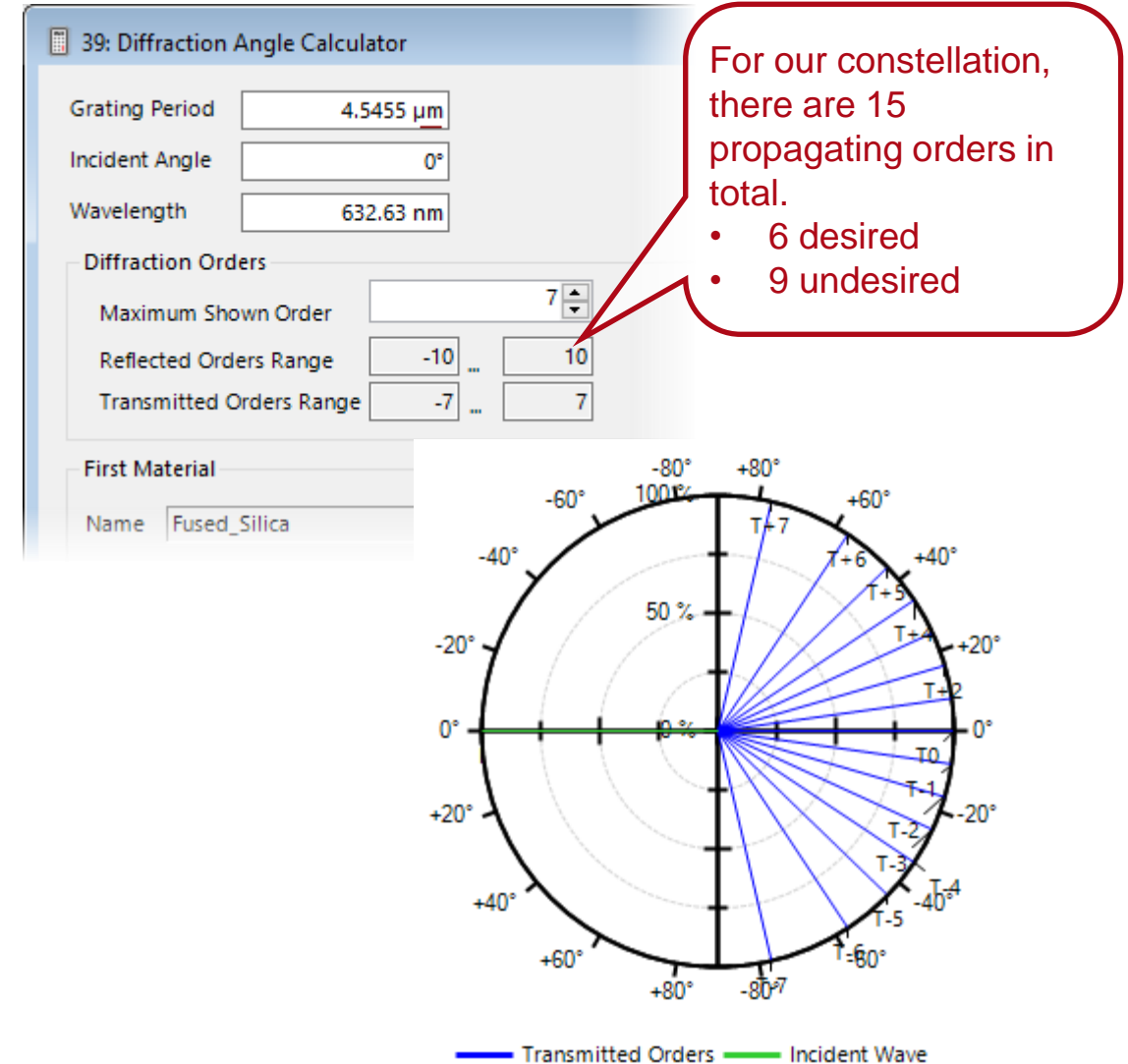
In this use case the following merit functions are defined:

1. Efficiency of Desired (Working) Orders
2. Uniformity Error of Desired (Working) Orders
3. Maximum Efficiency of Undesired Orders (excl. 0th)
4. Efficiency of Undesired 0th Order
5. Efficiency of Undesired Orders

The six desired (working) orders are: -5, -3, -1, 1, 3, 5

How many undesired orders are to be considered?

→ We used VirtualLab Fusion's *Diffraction Angle Calculator* to ascertain the number of propagating orders.





# Using the Programmable Grating Analyzer

The standard *Grating Order Analyzer* is a great tool for displaying all the efficiencies of interest in manifold ways.

But for defining arbitrary desired merit functions the *Programmable Grating Analyzer* is the most suitable tool.

The top part of the image shows a simulation schematic with three components: 'Ideal Plane Wave' (ID 0), 'Beam Splitting Grating' (ID 1), and 'Raw Data Detector' (ID 600). The grating is positioned at 'Z: 0 m' and the detector is also at 'Z: 0 m'. A red dashed line indicates a connection from the grating to the detector.

The middle part shows the 'Programmable Grating Analyzer' component (ID 802) in a library, with a red box around it and an arrow pointing to the 'Edit' button in the 'Edit Programmable Grating Analyzer' dialog box.

The 'Edit Programmable Grating Analyzer' dialog box has the following settings:

- Algorithm: Edit (button)
- Validity:
- Parameters:
  - Number\_of\_Desired\_Orders: 6
  - Total\_Number\_of\_Transmission\_Orders\_to\_Be\_Considered: 15
  - Visual\_Output

The bottom part shows the 'C# code of Programmable Grating Analyzer' in a source code editor:

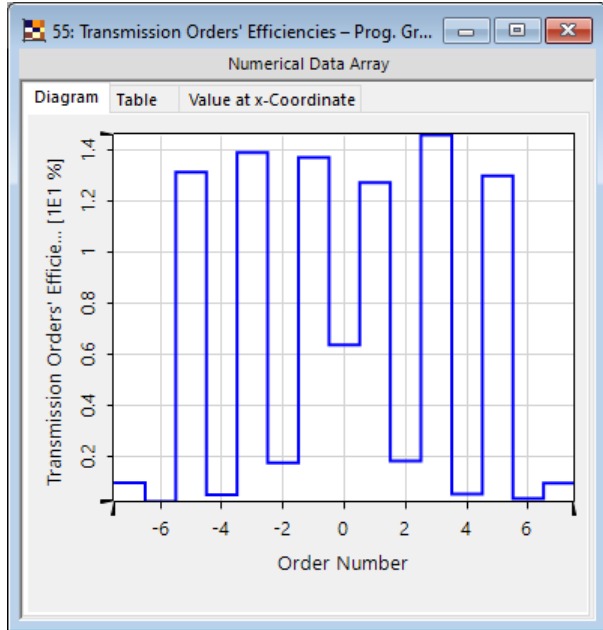
```
52 double minEffDesired = 1e-10;  
53 double maxEffDesired = 1e-10;  
54 List<int> desiredOrders = new List<int>();  
55 double maxEffUndesired = 1e-10;  
56  
57 for (int i = 0; i < Number_of_Desired_Orders; i++) { //only desired (working) orders  
    desiredOrderNumber = ((-1) * Number_of_Desired_Orders + 1) + 2 * i; // only odd orders: e.g. -5,-3,-1,+1,+3,+5  
    desiredOrders.Add(desiredOrderNumber);  
    OrderInfo desiredOrderInfo = TransmissionResults.GetOrder(desiredOrderNumber);  
  
    double effSingleDesired = 0;  
    if (desiredOrderInfo != null) {  
        effSingleDesired = desiredOrderInfo.Efficiency;  
    }  
    // #1: total efficiency of desired (working) orders  
    sumEffDesired += effSingleDesired;  
    minEffDesired = Math.Min(minEffDesired, effSingleDesired);  
    maxEffDesired = Math.Max(maxEffDesired, effSingleDesired);  
}
```

E.g. the uniformity error is evaluated based on the efficiencies of the desired orders according to:

$$UE = \frac{Eff_{max} - Eff_{min}}{Eff_{max} + Eff_{min}}$$

It's even possible to define a visual output from the *Programmable Grating Analyzer*. In order to get a nice spike diagram some manual adjustments are required, though.

# Rigorous Analysis of Initial Beamsplitter Design



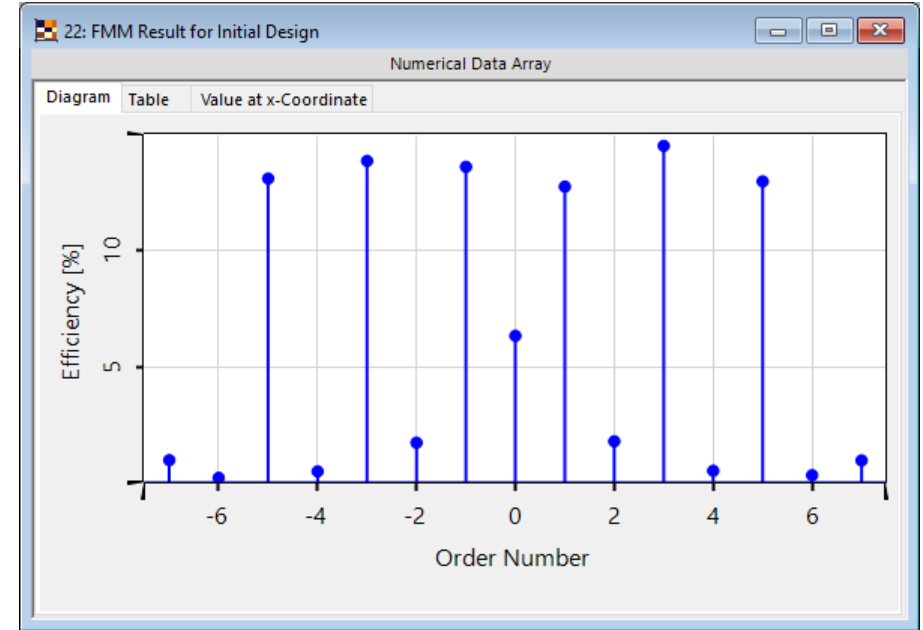
adjustments via *Property Browser*

Property Browser

55: Transmission Orders' Efficiencies – Prog. Grating An...

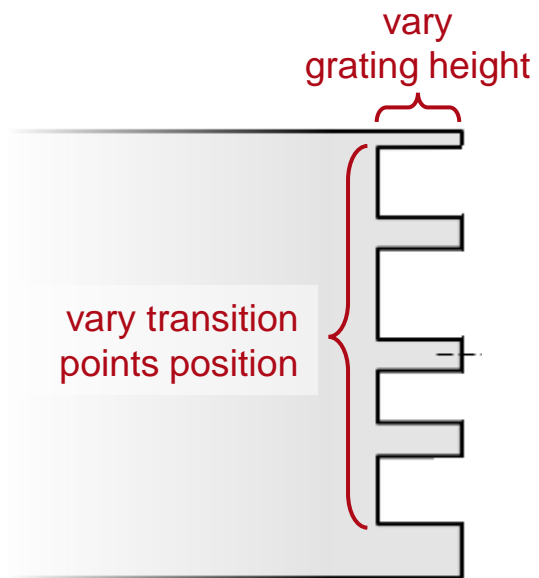
View Object Selections

- General
  - Window Size: 600, 420
  - Transposed View: False
  - Zoom Factor: 33.73333333
  - Zoom Factor Unit: 1 px / unity
- Data
  - Auto Scaling of Data: True
  - Minimum Number of y-Axis Ticks: 2
  - View Interpolation: No Interpolation
- Labels
  - Font Size: 10
- Lines and Symbols
  - Line Color: Blue
  - Line Thickness: 2
  - Symbol Color: Blue
  - Symbol Scaling Factor: 1.5
  - Symbol Shape: Filled Circle
  - Use Smoothed Graphics: True
- Selection (General)
  - Selection Mode: Range



Merit Function	Result
Efficiency of Desired (Working) Orders	80.9%
Uniformity Error of Desired (Working) Orders	6.8%
Maximum Efficiency of Undesired Orders (excl. 0th)	1.8%
Efficiency of Undesired 0th Order	6.4%
Efficiency of Undesired Orders	13.4%

# Set the Optimization Parameters



56: Parametric Optimization Document

**Parameter Selection**  
Select the parameters which shall be varied during optimization.

You can select one or more parameter which shall be varied within the optimization.

Filter by...  Show Only Varied Parameters

1	2	*	Object	Category	Parameter	Vary	Original Value
			"Beam Splitting Grating" (#1)	Stack #2 (Stack)	Surface #1 (Transition Point List Interface)   Scaling z-Direction	<input checked="" type="checkbox"/>	1
					Surface #1 (Transition Point List Interface)   Position of Transition Point # 2	<input checked="" type="checkbox"/>	-1.84175 $\mu\text{m}$
					Surface #1 (Transition Point List Interface)   Position of Transition Point # 3	<input checked="" type="checkbox"/>	-518.75 nm
					Surface #1 (Transition Point List Interface)   Position of Transition Point # 4	<input checked="" type="checkbox"/>	-250 $\mu\text{m}$
					Surface #1 (Transition Point List Interface)   Position of Transition Point # 5	<input checked="" type="checkbox"/>	431.25 nm
					Surface #1 (Transition Point List Interface)   Position of Transition Point # 6	<input checked="" type="checkbox"/>	1.75425 $\mu\text{m}$

specified free parameters for the optimization

# Two Optimization Processes for Comparison

In this use case we demonstrate two optimizations with differently configured aims and constraints:

- In **optimization #1**, the uniformity error is prioritized.
- In **optimization #2**, the 0th order should be minimized as well.

Concerning the merit function constraints, the user can specify

- what is the individual target value, range, lower or upper limit
- and via a weight, what the contributions of these should be.

"Programmable Grating Analyzer" (# 802)	Value #1: Efficiency of Desired	<input checked="" type="checkbox"/>	50	Target Value	0.9
	Value #2: Uniformity Error of	<input checked="" type="checkbox"/>	200	Target Value	0
	Value #3: Maximum Efficiency	<input checked="" type="checkbox"/>	0	Upper Limit	0.03
	Value #4: Efficiency of	<input checked="" type="checkbox"/>	500	Lower Limit	0.02
	Value #5: Efficiency of	<input checked="" type="checkbox"/>	0	Upper Limit	0.1

For the optimization, the inbuilt *Down-Hill Simplex* algorithm is applied.

Downhill Simplex	▼
Downhill Simplex	
Powell's Direction Set Method	
Levenberg-Marquardt Algorithm	

# Configuration of the Merit Function Constraints

## Optimization #1

### Contributions of Considered Merit Functions

Efficiency of desired Orders	0.2%
Uniformity Error of des. Ord.	99.8%

Constraint Host	Constraint Name	Use	Weight	Constraint Type	Value 1	Value 2	Start Value	Contribution
"Beam Splitting Grating" (# 1)	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	0.5	1.5	1	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-1.84175 $\mu\text{m}$	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-518.75 nm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-250 pm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	431.25 nm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	1.75425 $\mu\text{m}$	0 %
"Programmable Grating Analyzer" (# 802)	Value #1: Efficiency of Desired (Working) Orders	<input checked="" type="checkbox"/>	1	Target Value	0.9		0.8093572324	0.1983999243 %
	Value #2: Uniformity Error of Desired (Working)	<input checked="" type="checkbox"/>	1000	Target Value	0		0.06428818407	99.80160008 %
	Value #3: Maximum Efficiency of Undesired Orders	<input checked="" type="checkbox"/>	0	Upper Limit	0.03		0.01796733629	0 %
	Value #4: Efficiency of Undesired Zeroth Order	<input checked="" type="checkbox"/>	0	Upper Limit	0.03		0.06340706733	0 %
	Value #5: Efficiency of Undesired Orders	<input checked="" type="checkbox"/>	0	Upper Limit	0.1		0.1340061047	0 %
"Beam Splitting	Transition Point List Surface # 1   Minimum Feature	<input checked="" type="checkbox"/>	1	Lower Limit	300 nm		431 nm	0 %

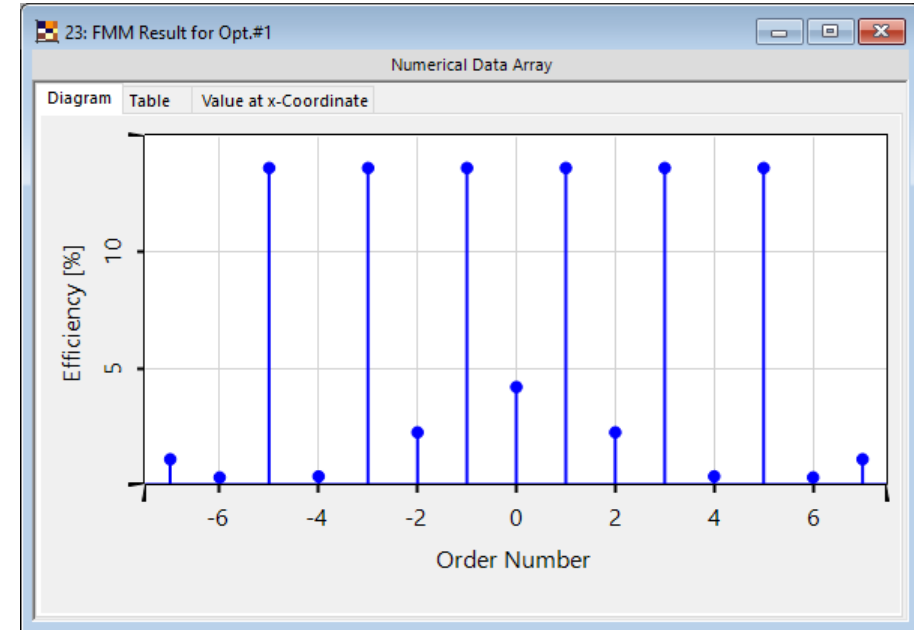
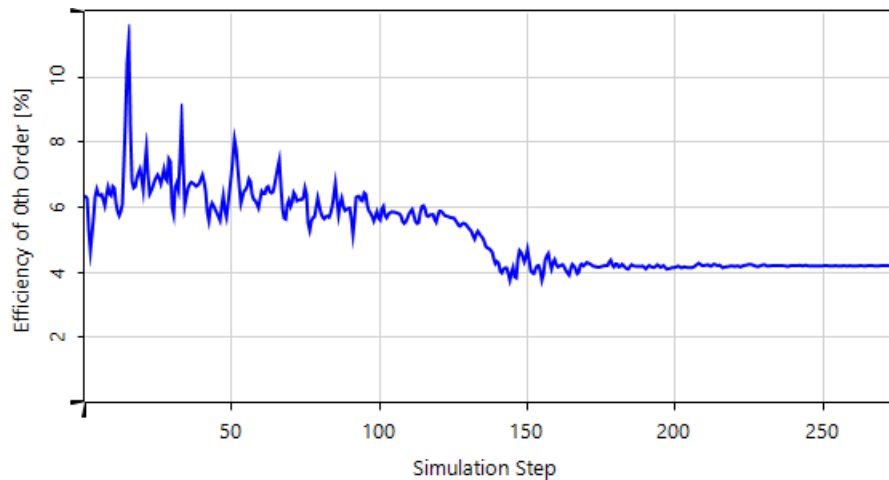
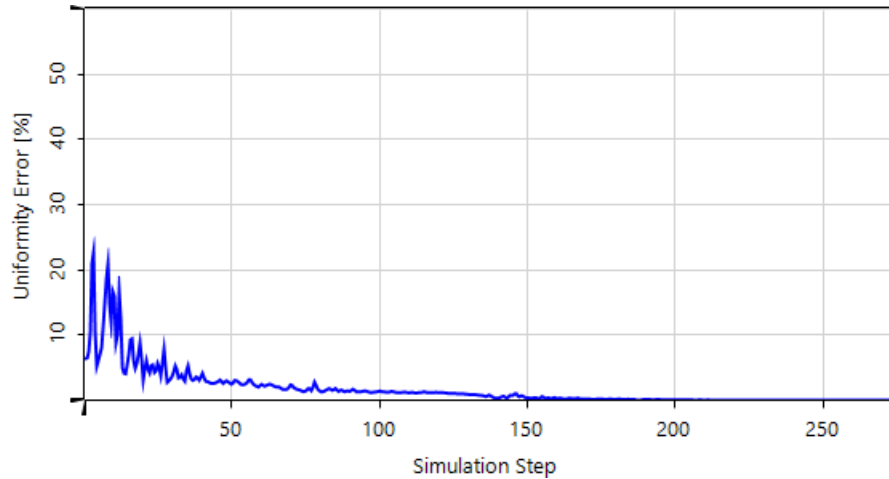
## Optimization #2

### Contributions of Considered Merit Functions

Efficiency of Desired Orders	18.8%
Uniformity Error of Des. Ord.	37.9%
Efficiency of Undesired 0th Ord.	43.2%

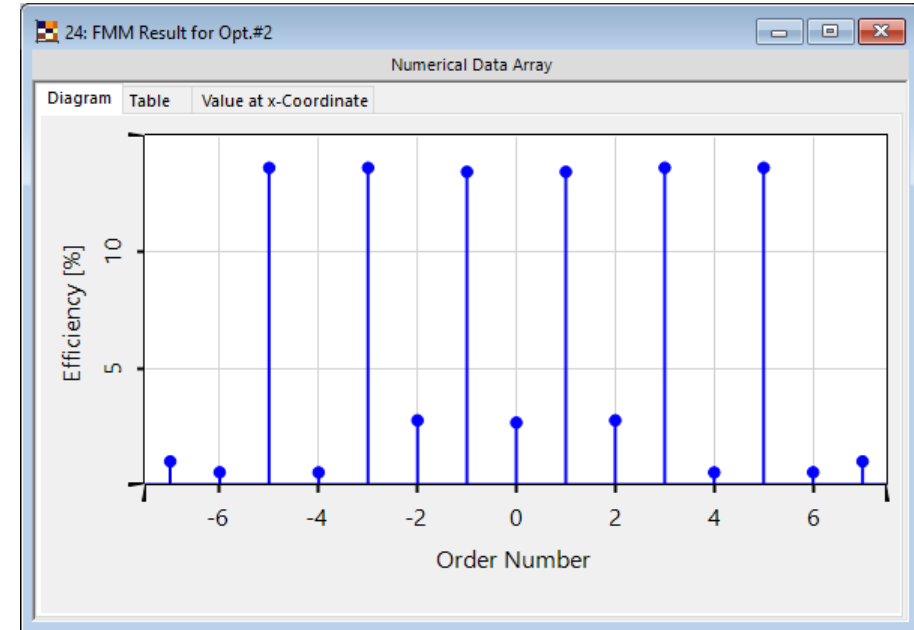
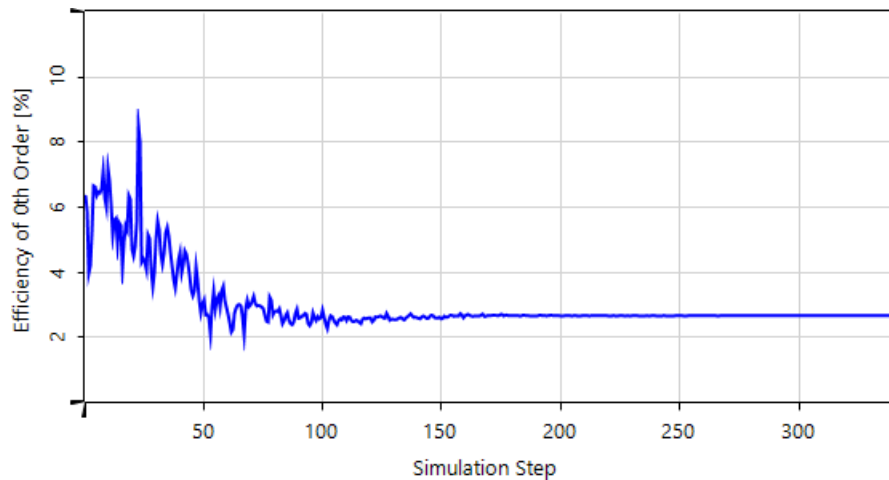
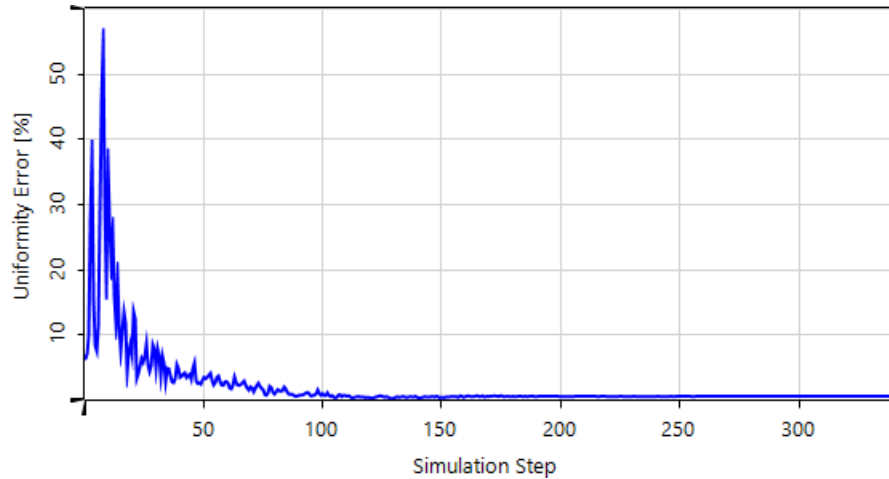
Constraint Host	Constraint Name	Use	Weight	Constraint Type	Value 1	Value 2	Start Value	Contribution
"Beam Splitting Grating" (# 1)	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	0.5	1.5	1	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-1.84175 $\mu\text{m}$	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-518.75 nm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	-250 pm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	431.25 nm	0 %
	Stack #2 (Stack)   Surface #1 (Transition Point List	<input checked="" type="checkbox"/>	1	Range	-2.27275 $\mu\text{m}$	2.27275 $\mu\text{m}$	1.75425 $\mu\text{m}$	0 %
"Programmable Grating Analyzer" (# 802)	Value #1: Efficiency of Desired (Working) Orders	<input checked="" type="checkbox"/>	50	Target Value	0.9		0.8093572324	18.84873241 %
	Value #2: Uniformity Error of Desired (Working)	<input checked="" type="checkbox"/>	200	Target Value	0		0.06428818407	37.9260962 %
	Value #3: Maximum Efficiency of Undesired Orders	<input checked="" type="checkbox"/>	0	Upper Limit	0.03		0.01796733629	0 %
	Value #4: Efficiency of Undesired Zeroth Order	<input checked="" type="checkbox"/>	500	Upper Limit	0.02		0.06340706733	43.22517139 %
	Value #5: Efficiency of Undesired Orders	<input checked="" type="checkbox"/>	0	Upper Limit	0.1		0.1340061047	0 %
"Beam Splitting	Transition Point List Surface # 1   Minimum Feature	<input checked="" type="checkbox"/>	1	Lower Limit	300 nm		431 nm	0 %

# Optimization #1 (Priority = Uniformity Error)



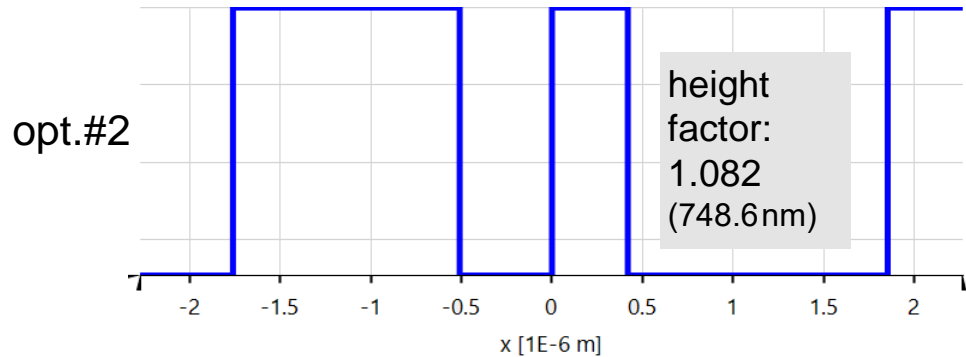
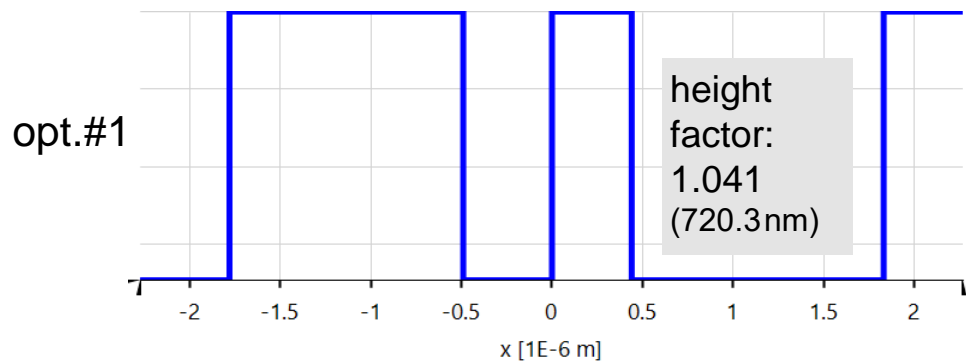
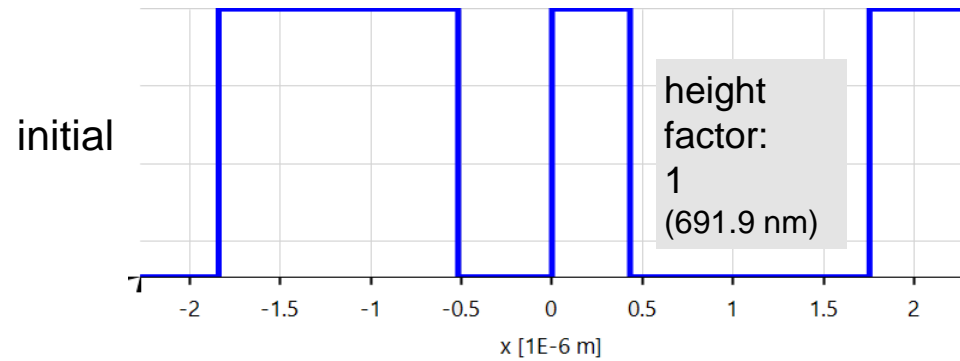
Merit Function	Result
Efficiency of Desired (Working) Orders	81.8%
Uniformity Error of Desired (Working) Orders	0.0%
Maximum Efficiency of Undesired Orders (excl. 0th)	2.3%
Efficiency of Undesired 0th Order	4.2%
Efficiency of Undesired Orders	12.3%

# Optimization #2 (Priority = Uniformity Error & Low 0th Order)



Merit Function	Result
Efficiency of Desired (Working) Orders	81.5%
Uniformity Error of Desired (Working) Orders	0.6%
Maximum Efficiency of Undesired Orders (excl. 0th)	2.8%
Efficiency of Undesired 0th Order	2.7%
Efficiency of Undesired Orders	12.3%

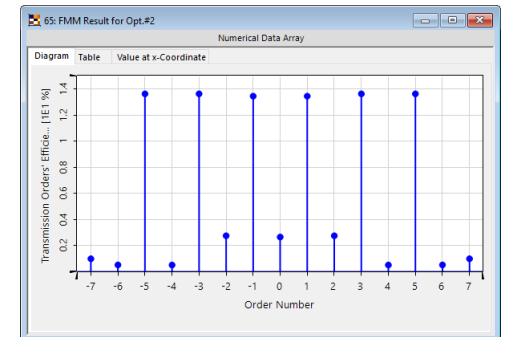
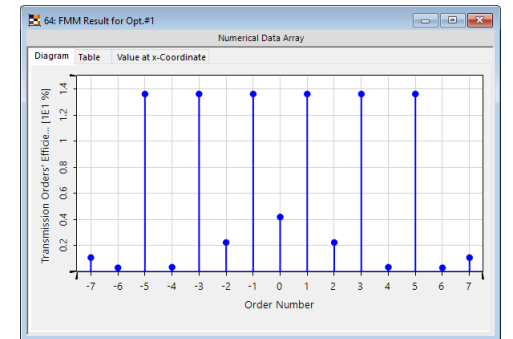
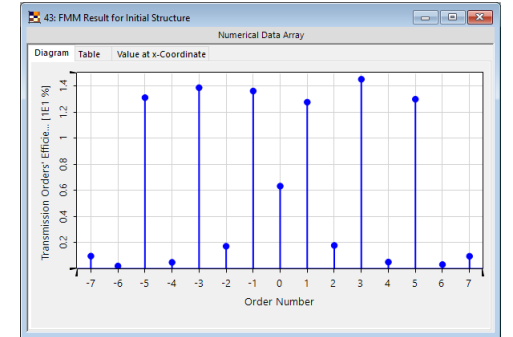
# Comparison of Rigorous Results (Initial – Opt.#1 – Opt.#2)



Merit Function	Initial	Opt.#1	Opt.#2
Total Efficiency	80.9%	81.8%	81.5%
Uniformity Error	6.8%	0.0%	0.6%
Max. Stray Light (excl. 0th)	1.8%	2.3%	2.8%
Efficiency of 0th Order	6.4%	4.2%	2.7%
Efficiency of Stray Light	13.4%	12.3%	12.3%

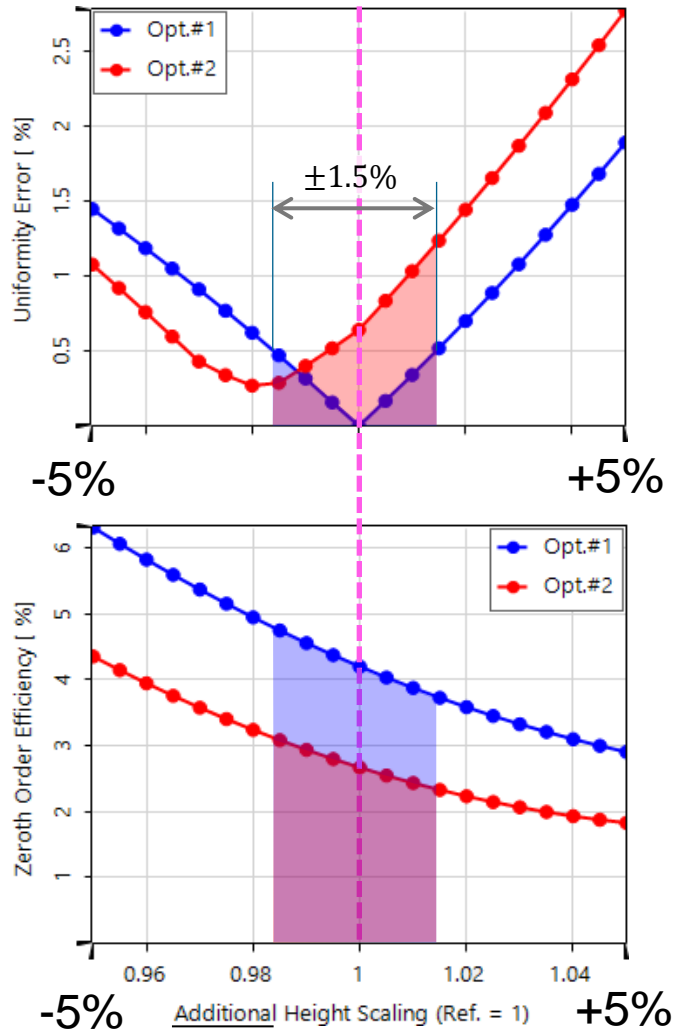
## Note

Since the rigorous results of a high-NA beam splitter might deviate considerably from the approximate results, consideration should be given to investigating and, if necessary, reoptimizing supposedly inferior initial designs.





# Results from Tolerance Simulations

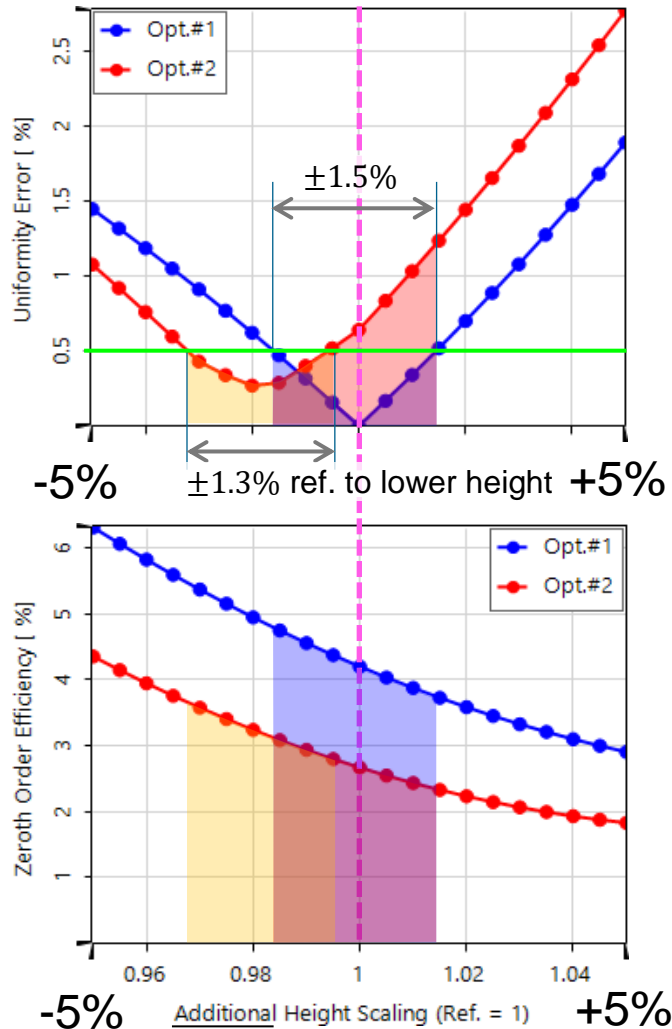


- It was investigated how the quality functions behave for possibly height tolerances during production in the range of  $\pm 5\%$ .
- In most parts of the tolerancing range regarding an etching depth error of approx.  $\pm 1.5\%$  (length of blue & red areas), the design from the 2<sup>nd</sup> optimization exhibits a distinctly worse uniformity.
- At first glance, it may seem strange for optimization #2 the minimal uniformity error (red curve @0%) is not centered. This is because low 0<sup>th</sup> order efficiency was prioritized in optimization #2 and some uniformity was sacrificed to achieve this aim.
- As a result, the structure of the 2<sup>nd</sup> optimization has always a distinct lower 0<sup>th</sup> order for the whole envisaged range of the tolerancing analysis.

**Note:**

The reference value 1 in the tolerance simulation results always refers to the individually optimized height of the examined structure (indicated by the purple line).

# Conclusion from Tolerance Simulations



- Tolerance testing provides a better information base for deciding what is the most suitable structure for the desired application.
- It can be seen, that the structure of the 2<sup>nd</sup> optimization yields uniformity errors below 0.5% (green line) over a similar tolerancing range of  $\pm 1.3\%$  (length of yellow area) if the height with the lowest uniformity error is used.
- Thus, the 2<sup>nd</sup> optimization result with an additional height scaling of 0.9825 (707.7 nm) might pose a good solution with an overall suitable performance. In below table the according results are shown in the last column titled "Opt.#2b".

Merit Function	Initial	Opt.#1	Opt.#2	Opt.#2b
Total Efficiency	80.9%	81.8%	81.5%	81.6%
Uniformity Error	6.8%	0.0%	0.6%	0.2%
Max. Stray Light (excl. 0th)	1.8%	2.3%	2.8%	2.6%
Efficiency of 0th Order	6.4%	4.2%	2.7%	3.2%
Efficiency of Stray Light	13.4%	12.3%	12.3%	12.3%

# Document Information

title	High NA Beam Splitter Optimization with User-Defined Merit Functions
document code	DOE.0005
document version	1.0
software version	2021.1 (Build 1.180)
software edition	VirtualLab Fusion Advanced
category	Application Use Case
further reading	<ul style="list-style-type: none"><li>- <a href="#"><u>Programmable Grating Analyzer</u></a></li><li>- <a href="#"><u>Design and Rigorous Analysis of Non-Paraxial Diffractive Beam Splitter</u></a></li><li>- <a href="#"><u>Design of Diffractive Beam Splitters for Generating a 2D Light Mark</u></a></li></ul>